COURSES FOR 2019/2020 ACADEMIC YEAR

Course	Credits	Course Description
FIRSTSEMESTER		
Skills Courses (Core)		
Block 1 (26 August - 13 September 2019)		
MSCI 501/601	2	Computing and LaTeX
MSCI 503/603	2	Introduction to Python and SciPy
MSCI 521/621	2	Introduction to Linear Algebra
Block 2 (16 September - 4 October 2019)		
MSCI 505/605	2	Mathematical Problem Solving
MSCI 563/663	2	Introduction to Probability Theory
E - week (7 - 11 October 2019)		
Block 3 (14 October - 1 November 2019)		
MSCI 507/607	2	Concepts in Physics and Physical Problem Solving
MSCI 553/653	2	Ordinary Differential Equations
Reading week (4-8 November 2019)		
Review Courses (Electives)		
Block 1 (11 - 29 November 2019)		
MSCI 569/669	2	Statistical Methods in Climate Science
MSCI 538/638	2	Introductory Real Analysis
MSCI 573/673	2	Complex Networks with Computations

Block 2 (2 - 20 December 2019)		
MSCI 571/671	2	Complex Analysis with Applications
MSCI 541/641	2	Advanced Numerical Analysis and Scientific Computing with Python
MSCI 523/623	2	Algebraic Number Theory
SECONDSEMESTER		
Review Courses (Electives) continued.		
Block 3 (6 - 24 January 2020)		
MSCI 556/656	2	Numerical Methods for Climate Science
MSCI 533/633	2	Functional Analysis
MSCI 572/672	2	Quantum Theory in a Nutshell
Block 4 (27 January - 14 February 2020)		
MSCI 522/622	2	Introduction to Topology and Knot Theory
MSCI 578/678	2	Introduction to General Relativity
MSCI 568/668	2	Data Visualisation
Reading week (17-21 February 2020)		
Block 5 (24 February - 13 March 2020)		
MSCI 596/696	2	Mathematical Epidemiology
MSCI 576/676	2	Introduction to Quantum Field Theory
MSCI 532/632	2	Noncommutative Algebra and Geometry
MSCI 594/694	2	Stochastic Analysis
Block 6 (17 March - 4 April 2020)		

MSCI 562/662	2	Pattern Recognition and Machine Learning
MSCI 558/658	2	Dynamical Systems
MSCI 524/624	2	Introduction to Differentiable Manifolds
Research Phase (Core) (4 April - 10 June 2020)		
MSCI 500/600	6	Research Project

SKILLS COURSE DESCRIPTIONS

MSCI 501/601: Mathematical Preliminaries and LaTeX
Des Johnston, Department of Mathematics, Heriot-Watt University, Edinburgh UK

This course introduces the AIMS computing facilities, both local Ubuntu laptop logons and limits, and Google Apps for Education services, with the associated Acceptable Use Policies. It then continues with an exploration of the Ubuntu desktop and Google Apps services, touching on style, efficiency, and netiquette, during the first few days.

The main part of the course consists of an introduction to LaTeX (using texmaker), followed by working through the book: `http://en.wikibooks.org/wiki/LaTeX`, and a few associated documents such as the American Mathematical Society Short Math Guide about using the AMS packages with LaTeX. The course follows a style where students read a chapter and ask questions or demonstrate a new piece of information in the next class. After this course, students should be able to complete their AIMS essay professionally, finding and using LaTeX resources in the internet independently. The rest of the time is spent on general queries about the Ubuntu desktop, Google Apps, and using various science software, interpreters, or compilers via a simple text editor and terminal, perhaps a language specific interpreter, and optionally a full front-end for some languages (notebook for **SAGE**, **rstudio** for **R**).

Learning objectives are that a student should have significant confidence in Ubuntu, Google Apps, LaTeX, and should understand some similarities between different scientific application interfaces, in order to complete an AIMS essay easily and efficiently, as regards general computing and typesetting.

MSCI 503/603: Introduction to Python and SciPy
Eyram Schwinger, Univeirisity of Ghana, Legon, Ghana

The course covers (1) software development for (2) scientific applications in (3) Python. The course assumes limited programming background, but familiarity with general maths concepts (e.g., calculus, probability, and geometry) and basic numerical mathematics (e.g., finite difference methods).
The course will focus on the main tasks in scientific computation:
- reading / writing data
- transformation / subsetting of data
- bulk analysis of data
- approximation and simulation of systems
- visualization

To support accomplishing these feats, the course will introduce students to a variety of programming constructs:
- variables, constants, input and output devices
- flow control with branching (if-else) and iteration (for, while)

functions
- collections
- functional programming (comprehensions, iterators, generators)
objects
- reuse / libraries / packages
- several development tools and practices that are especially important to scientific reproducibility:
- version control / data management
- unit testing and literate programming

MSCI 521/621: Introduction to Linear Algebra
Angela Tabiri, AIMS-Ghana

In this course, we will study vectors, matrices, linear transformations, systems of linear equations and eigenvalues and vectors. For vectors, we will introduce the concepts of linear independence, span, bases and subspaces. Under matrices, the focus will be on operations of matrices, special matrices, elementary row operations and the rank of a matrix. Properties of linear transformations and the rank-nullity theorem will be discussed. Given a system of linear equation, we will learn how to write it in terms of matrices and how to solve the system of equations using elementary row operations. We will also learn how to find eigenvalues and vectors and why they are important.

MSCI 505/605: Mathematical Problem Solving
Bernard Oduoku Benson, Department of Mathematics, KNUST, Kumasi, Ghana

This course considers a variety of elementary, but challenging problems in different branches of pure mathematics. Investigations, comparisons of different methods of attack, literature searches, solutions and generalizations of the problems will arise in discussions in class. The objective is to illustrate, and gain practical experience of, different approaches to problem solving and research.

MSCI 563/663: Introduction to Probability Theory
Wolfgang König and Benedikt Jahnel, Weierstrass Institute, Berlin, Germany

This will be an introduction to the mathematical treatment of the theory of probabilities on base of discrete models and on probabilities with Riemann-densities. We start with elementary probability spaces, the concept of independence, random variables, expectations and variances, and then we proceed with Poisson limit theorem, distributions on the integers and their generating functions, the most important distributions and their properties (Poisson, Binomial, exponential, normal). Finally, we consider distributions of sums of independent random variables and the weak law of large numbers, fundamental inequalities (Markov and Chebychev), and stop at the central limit theorem.

This theoretical material will be accompanied with examples and exercises. The material is fundamental and may be found in many textbooks. The prerequisites are Analysis 1 and Linear Algebra 1; usually such a course is attended by second-year math students.

MSCI 507/607: Concepts in Physics and Physical Problem Solving Laure Gouba, Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy

The course introduces first vectors, unit systems and a review of fundamental physics. Topics include classical mechanics and electromagnetism. The course is interactive through discussions and the problems solving sessions are to be done in small groups.

MSCI 553/653: Ordinary Differential Equations Patrick Dorey, Department of Mathematical Sciences, Durham University, UK

This course examines the theory and applications of differential equations, covering the following topics: classification and elementary methods of integration; existence and uniqueness results for systems of ODEs; applications of second order systems; phase diagrams, equilibrium points and the concept of stability. The course will include plenty of exercises and one project to be carried out in small groups.

AIMS African Institute for Mathematical Sciences GHANA

REVIEW COURSE DESCRIPTIONS

MSCI 569/669: Statistical Methods in Climate Science

Bright Emmanuel Owusu, Department of Mathematics, Presbyterian University College, Ghana.

Objectives:
This course is to introduce graduate students to basic statistical concepts and reasoning relative to climate science. It will also help the students to gain experience in using appropriate statistical methods for the analysis of climate observations (data). Besides, the course will offer students practical insight into statistical tools that are suitable for climate-change modeling. By so doing, the student will be educated on how to specify, fit and interpret numerous statistical models and applied them to answer scientific questions patterning to climate change in R statistical software and Instat climatic software.

After the course, students will:

- Exhibit in-depth understanding of the mechanisms and processes behind climate change
- Demonstrate the ability to infer and dispassionately assess estimates of future climate and its effects
- Describe the main concepts in statistical climate science
- Select and compare appropriate statistical methods for climate data analysis and to draw valid inferences.
- Apply statistical methods in climate science in R programming language and Instat.
- The skills acquired can be used in geosciences

Content:
The lecture content covers

- Introduction/Overview of the climate system The climate system, past climate change, the evidence for the current change
- Exploratory climate data analysis
- Probability distributions for climate data
- Statistical inference and testing
- Linear modeling
- Logistic regression
- Non-Parametric tests, Mann-Kendall test, Sen's slope test
- Multivariate methods, Factor Analysis, Empirical Orthogonal functions

MSCI 538/638: Introductory Real Analysis, Prof. Francis Oduro, AIMS-Ghana

The course content includes

- Defining Properties of the Real Number System
- Metric Space properties in $\mathbb{R}$
- Limits, Continuity and Differentiability (Hahn-Banach Theorem)
- Riemann Integral and its properties
- Sequences of Real Numbers
- Sequences of Functions
- Extensions to $\mathbb{R}^n$

MSCI 573/673: Complex Networks with Computations Philip Knight, Department of Mathematics and Statistics, University of Strathclyde, Glasgow, Scotland

General Aims
- To demonstrate the central role network theory plays in mathematical modeling.
- To show the intimate connection between matrix algebra and graph theory and to use this connection to develop a practical approach to analysing networks.
-To understand and use numerical techniques to extract useful information from the structure of real-life networks.
Outline Syllabus
- Introduce/review basic definitions and theory of networks/graphs with an emphasis on matrix algebra: including degree and degree distribution, trees, cliques, bipartite graphs, spanning trees, graph Laplacian, spectral properties of networks.
- Describe the theory and implementation of some modern algorithms in numerical linear algebra, particularly for eigenvalue computation and evaluating matrix functions, with a view to applications in complex networks: subspace methods for eigenvalue computation; equivalent definition of matrix functions; matrix powers and the matrix exponential; computation of centrality measures; PageRank.
- Brief historical introduction to applications of complex networks.
- Analysis of some modern applications of complex networks. E.g., social networks, internet, transportation networks, biological (inter-species and intra-cellular). Exploit the computational methods previously introduced to calculate relevant measures of the network.

MSCI 571/671: Complex Analysis with Applications
Douglas Adu-Gyamfi, Department of Mathematics, University of Ghana

This course begins with the properties of complex numbers. We examine functions of a complex variable; analytic and holomorphic functions. Then we turn to complex integration, Cauchy's integral theorem and Cauchy's integral formulae. We finish with the residue theory and applications.

MSCI 541/641: Advanced Numerical Analysis and Scientific Computing with Python
Georg Bader, Brandenburg University of Technology Cottbus - Senftenberg, Germany

Methods for the numerical simulation of mathematical models have been the focus of intensive research for well over 50 years, and the demand for better and more efficient methods has grown as the range of applications has increased. Mathematical models involving partial differential equations (PDEs) as well as ordinary differential equations (ODEs) arise in diverse applications such as fluid flow, physics-based simulation, mechanical systems, earth sciences, and mathematical finance.

The course gives a general introduction to differential equations with the main focus on partial differential equations. Additionally, it is shown how to solve those numerically with the finite difference methods and/or the finite volume method.

We will use the freely available software Python, since it give us the opportunity to integrate realistic problem solving aspects into classical courses on Advanced Numerical Analysis. This provides the students with a new dimension for the analysis and understanding of complex problems through a methods of numerical approximation and visual respresentation of the solution.

Precisely, the course covers a selection of general purpose numerical methods, both from the mathematical analysis as well as from the practical problem solving aspect.

MSCI 523/623: Algebraic Number Theory
Jeffery Ezearn, Department of Mathematics, KNUST, Kumasi, Ghana

Algebraic Number Theory is the study of finite extensions of the rational field by using techniques of abstract algebra (mostly commutative algebra and finite group theory). It has application in other areas of mathematics including algebraic geometry and cryptography. This course provides an introduction to algebraic number theory concentrating on the study of the ring of integers of number fields and the ramification of their primes. Only basic algebra is assumed.

Sage will be used to help students to explicitly understand the structure of number fields and the factorization of primes in an extension field. The course will help students to improve their ability to skilfully conceptualize, analyze and evaluate a given maths theoretical framework, as well as their ability to handle data by using theoretical framework.

MSCI 556/656: Numerical Methods for Climate Science
Mbehou Mohamed, Department of Mathematics, University of Yaounde, Cameroon

The aim of the course is to introduce students to the computational techniques needed to solve physical problems arising from climate sciences and to develop the programming skills necessary to implement the techniques. They will implement some of these methods using the Python/Matlab programming languages.

By the end, students should be able to develop numerical algorithms for solving basic differential equations commonly encountered in these fields and implement them as computer programs. Students should also be able to use numerical analysis to evaluate the results produced by the programs and design ways to improve them.

The content of the course is as follows:

1. Finite difference methods for PDE
   - Finite differences and Taylor series expansions
   - Derivatives with variable coefficients
   - Finite difference examples : 1D explicit heat equation, Energy Balance Model (EBM),...
   - Implicit FD schemes and boundary conditions
2. Non-linearities with FD methods, Example of Shallow Water Equation
3. Two-dimensional equation with FD, Advection equations with FD
4. Finite Volume Methods

MSCI 533/633: Functional Analysis,
Jeffery Ezearn, Department of Mathematics, KNUST, Kumasi, Ghana

Normed linear space, Banach space, Bounded linear maps, Inner product space, Hilbert space, Projection theorem, Riesz representation theorem; Measurable sets, Lebesgue Integral, L_p spaces; Sobolev spaces.

MSCI 572/672: Quantum Theory in a Nutshell
Michael Scherer, Institute for Theoretical Physics, University of Cologne, Germany

In this course I will teach the basics of Quantum Theory. The main focus will be the introduction of the essential concepts in Quantum Theory and to substantiate them with their mathematical formulation. The most important ideas which I will discuss are

(1) the superposition principle,

(2) particle-wave duality,

(3) the uncertainty principle,

(4) the quantization of energy levels and

(5) the existence of zero-point energy.

In particular, I will discuss the physics and mathematical formalism of two-level systems, thoroughly, to establish a sound basis for a possible follow-up lecture course on quantum information. In order to consolidate the understanding of these ideas, I introduce basic examples for physical phenomena that clarify these concepts. Additionally, I will let the students calculate instructive examples in class themselves. I will also offer short projects for more advanced students.

MSCI 522/622: Introduction to Topology and Knot Theory Nezhla Aghaei, Institute for Theoretical Physics, Albert Einstein Center for Fundamental Physics, University of Bern, Switzerland

Course description:
This course can be useful for the master student in Mathematics, Physics and/or Computer science. Knots have been elements of human culture since the beginning of History. The first uses of knotting were arguably made by primitive societies still in the Paleolithic Era and knots have been represented in artistic depictions of everyday life since ancient civilizations, like the Egyptian. Even in current times, knots appear in the most diverse forms and contexts, from elaborate artisanal braiding to magic tricks and puzzles. In climbing, for instance, stopper knots might be used to prevent the sliding of a rope.

The Mathematical study of knots started in the 19th century when Gauss developed a way of calculating the linking number of two knots, a quantity that indicates intuitively the amount of times the knots wind around each other. In that century, connections between the description of fluid mechanics and electromagnetism were starting to be made, thanks to the description of the latter in terms of field lines. In the past 50 years, knot theory has become an extremely well-developed subject. But there remain several notoriously intractable problems about knots and links, many of which are surprisingly easy to state. The focus of this course is this elementary aspect to knot theory. Our aim is a brief survey of what is known. The problems that we will concentrate on are the non-technical ones, although of course, their eventual solution is likely to be sophisticated. In fact, one of the attractions of knot theory is its extensive interactions with many different branches of mathematics: 3-manifold topology, hyperbolic geometry, Teichmüller theory, gauge theory, mathematical physics, geometric group theory, graph theory and many other fields.
Course strategy:
The contents of the course will be adjusted to the students? foreknowledge. We rather have the students understand a few concepts really well than just having heard of many concepts but not understanding them. To make the students well familiar with the concepts, we will ask them to do many small calculations themselves during the lecture.The course is intended to convey the important aspects of the sciences: So adding to the theoretical concept we use some objects to explain them better.

Depends on the background of the student we can invest more time at the beginning to understand the basics or putting more time to the further application of in physics or advance mathematical topics at the end.

Week one: What is knot theory/Redemeister moves/Types of knots/ Knots in biology, Chemistry and physics.

Week two: Invariants of Knots/ Surfaces and Knots/Links and graphs.

Week three: Polynomials/introduction to topology/3-manifolds topology.

Topics for advanced students:

- Relation of topology and high energy Physics
- Higher dimensional knots
- Quantum invariants of knots

MSCI 578/678: Introduction to General Relativity
Marc Casals, Brazilian Center for Research in Physics (CBPF), Brazil

LIGO recently detected gravitational waves (GWs) emitted by inspirals of black holes, marking the start of GW astronomy. In this course, within the framework of General Relativity, we will study black holes, the geodesic motion of particles around them, the wave equation obeyed by linear fields and, using mathematical software, we will calculate the so-called quasi-normal modes. These modes are characteristic frequencies of black holes and determine the GW at late times.

The course will consist of both theoretical lectures and hands-on exercises, and it will combine a range of skills, from mathematical to numerical (including the use of mathematical software), depending on the background and interests of the students.

I have lectured mini-courses on this and similar topics in various academic institutions. In particular, in November'2016 I lectured at the California Institute of Technology, U.S.A. Please see http://www.cbpf.br/ mcasals/Lectures.html.

MSCI 568/668: Data Visualisation
Craig Anderson and Manuele Leonelli, School of Mathematics and Statistics, University of Glasgow, UK

This course will enable students to develop advanced expertise in data analysis for a wide variety of subject areas (using R) and also understand the importance of data visualisation in uncovering any hidden structure in the data.

In the three weeks that the course will be taught we will explore data visualisation techniques, generalised linear models, and mixed effect models. The course will emphasise on interactive data visualisation techniques that are available within R and will use a range of displays to explore data.

The course makes extensive use of real data examples and emphasizes the role that statistical models play in addressing scientific questions and how these are translated into relevant statistical questions. Students will learn to distinguish between problems of parameter estimation, hypothesis testing and prediction.

It's important to note that the course will be accessible to students with no prior background but at the same time it will be challenging for students that may have seen some of these techniques before.

MSCI 596/696: Mathematical Epidemiology
Prof Francis Benyah, Department of Mathematics University of Cape Coast, Ghana

Many diseases are contagious, that is, you get them from someone who is already infected. Contagious diseases such as smallpox, polio, and plague are severe and even fatal; while the common cold and childhood illnesses like measles, mumps, rubella and chicken pox are usually relatively mild.

You can catch a cold over and over again, but you get measles, (or chicken pox) only once, because once you recover them, you acquire immunity against re-infection. Infectious disease like HIV/AIDS has neither a cure nor a vaccine, but it is preventable. Vector-borne diseases like Malaria, Dengue-fever and Rift Valley fever are transmitted not directly from an infected person, but rather from a vector, the mosquito.

Understanding the transmission characteristics of infectious diseases can lead to better approaches to controlling, and possibly eliminating them. In fact, many infectious diseases are eradicable, unfortunately, they remain endemic in society because of lack of proper control policy.

Mathematical models have become important tools for formulating and analyzing the transmission and control of infectious diseases. Such models, together with the estimation of key model parameters from data, as well as computer simulations, can lead to better approaches to controlling the transmission and possibly, eliminating these diseases.

This course introduces students to

- deterministic compartmental models in epidemiology,
- stability analysis of equilibrium points,
- determination of model parameters from data, and finally,
- simulation.

MSCI 576/676: Introduction to Quantum Field Theory,
Daniel Wohns and Gang Xu, Perimeter Institute for Theoretical Physics, Canada

Introduction to canonical quantization of scalar field theories. Classical field theory, special relativity, and quantum mechanics will be introduced or reviewed as necessary. The course will focus on conceptual issues and will include many simple examples.

MSCI 532/632: Noncommutative Algebra and Geometry
Ulrich Krähmer, Institute for Geometry, Technical University, Dresden

This will be an elementary course on Hopf algebras. Depending on the background of the audience, a possible aim is the definition of invariants of knots using the representations of quasitriangular (braided) Hopf algebras.

MSCI 594/694: Stochastic Analysis
Olivier M. Parmen and Moustapha Dieye, AIMS-Ghana

Course Content:
The aims at giving basic foundation on stochastic analysis. It will introduce the students to stochastic analysis with emphasis on Brownian motion, Ito's integral, Ito's formula and applications and stochastic differential equation. The theoretical results will be illustrated with numerical examples. If time allows, applications will be made to mathematical finance.

Mathematical Preliminaries: Measure theory, probability space, random variable, expectation, filtration, conditional expectation, martingales, stopping time.

Stochastic integral: Brownian motion and properties, construction of Ito's integral, properties of Ito's integral, extension of Ito's integral, Ito's formula and applications, Girsanov Theorem, Martingale representation Theorem.

Stochastic differential equations: Global existence and uniqueness; diffusions and the PDE connection; Feynman-Kac representation.

Application to Mathematical Finance: risk-neutral pricing; derivative securities.

MSCI 562/662: Pattern Recognition (Machine Learning)
Alessandro Crimi, INRIA-France (National French Institute for Research in Computer Science and Control) avenue du général Lecrerc Campus, Universitaire de Beaulieu, Cedex, France

Pattern recognition (also known as Machine Learning or Statistical Learning) is a scientific discipline involving design and development of statistical algorithms in order to understand behaviors based on empirical data. The general aim of these statistical techniques is to automatically 'learn' to recognize complex patterns and make intelligent decisions based on data. Nowadays, pattern recognition is used in several fields such as bioinformatics, speech recognition, finance, medical diagnosis and artificial vision.

An example of pattern recognition is classification. It attempts to assign an unknown input data to one of the given classes, e.g. determine whether a given unknown DNA sequence is more similar to another already known one, or whether a brain lesion in an

MRI will continue to grow or not. Other examples are regression and data dimensionality reduction (generally taught in the specific course 62H25).

The course will have a problem-based structure for each lesson as follows: - A challenging problem from one of the application fields is presented to stimulate the interest. - The theoretical aspects related to the mathematical formulation are taught (no assumption on the background is made). Related computer projects are investigated using SciPy and NumPy or their extensions due to the background of the lecturer, essays and computer projects will be related to the medical field.

Teaching material:

1. R.Duda, P.Hart "Pattern Classification" 2nd Ed.
2. C.Bishop "Pattern recognition and machine learning"
3. Notes from the Lecturers The lecturer has previously co-taught and taught a course called "Medical Imaging" at the University of Copenhagen, where pattern recognition was presented related to medical image analysis.

MSCI 558/658: Dynamical Systems
Djoko Wirosoetisno, Durham University, UK

Nonlinear differential equations and dynamical systems is a vast area and practitioners include applied mathematicians, analysts and others in science and engineering. Although there are many books on nonlinear science, they are not always accessible to beginning graduate students as they often require extensive mathematical preparation. The main aim of this course is to provide a broad education in the area that is mathematically insightful yet devoid of extensive formalism. The main topic will be a study of of the qualitative and geometric theory of nonlinear differential equations and dynamical systems. The approach taken will depend heavily on examples. Students will

(a) Learn a number of techniques which will increase their chances of success when faced by a nonlinear problem.

(b) Be provided with the fundamental ideas of the subject so that they will find some of the more advanced textbooks accessible.

MSCI 524/624: Introduction to Differentiable Manifolds
Casim Abbas, Department of Mathematics, Michigan State University, USA

The concept of a differentiable manifold plays a fundamental role in geometry and many areas of physics. Basic topics are: Differentiable manifolds, Sard's Theorem, Transversality, Vector Fields, Tangent and cotangent bundle, Differential Forms, Orientation and Integration on manifolds. Optional topics are: Maxwell's equations, the formalism of Hamiltonian mechanics, Intersection Number, Poincare-Hopf Theorem, Euler Characteristic, Gauss-Bonnet Theorem