Chaos, Solitons and Fractals 145 (2021) 110801

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Review

Controlling crime with its associated cost during festive periods using mathematical techniques

Nicholas Kwasi-Do Ohene Opoku[a,b,*], Georg Bader[c], Edem Fiatsonu[a]

[a] African Institute for Mathematical Sciences, Biriwa, Ghana
[b] Kwame Nkrumah University of Science and Technology, Kumasi, Ghana
[c] Brandenburg Technical University Cottbus-Senftenberg, Germany

ARTICLE INFO

Article history:
Received 4 August 2020
Revised 11 February 2021
Accepted 17 February 2021

Keywords:
Crime
Festive-periods
Basic-reproduction-number
Bifurcation
Optimal-control
Cost-effectiveness

ABSTRACT

In this paper, we seek to control crime at its minimal level during festive periods such as Christmas, Valentine's day and entertainment events such as music awards. We used epidemiological-borrowed concepts to understand and model the dynamics of crime during these periods. We analyze the fundamental properties of the model, compute the crime basic reproduction number, R_0, using the next generation matrix approach and use the output to establish the steady states of the model. The crime-free steady state is found to be locally stable whenever $R_0 < 1$. The center manifold theorem is used to show the existence of bifurcation at $R_0 = 1$. The model is then transformed into an optimal control problem with three control interventions (education, detention and sacking) to obtain the best strategy to control crime at its minimum level. The control reproduction number was determined to show scenarios when the implementation of the controls give an $R_0 < 1$ or $R_0 > 1$. Moreover, numerical simulations are carried out to affirm the theoretical properties of the model. From the simulations, education is observed to be the best single strategy to apply but, alternatively, incorporating two or more control interventions equally give a better result. Finally, cost effectiveness analysis was employed on the control strategies and it shows that education and sacking are the most cost-effective strategies to minimise crime during events.

1. Introduction

Entertainment events during holidays, festive occasions (such as Christmas, Valentine days etc.) are periods that raise problems for the general public since criminals see these periods as a chance to commit criminal offences. According to [1], three key elements that constitute a crime; [1]crime as a criminal act (an unlawful bodily movement where the defendant voluntarily act in a way that poses a considerable threat of personal injury or injury to others), [2]crime is taken into account as an omission of acts when someone fails to perform a duty by law and [3]crime as a criminal intent where criminal acts are administered with a culpable mind. Thus, the European Union Statistics put crime in six categories which reflect the range of policy and legal systems, that is homicide, violent crime, robbery, domestic burglary, theft and drug trafficking [2].

Crime has been widely studied as a serious sociological problem that has been interpreted as bad conduct within the prescientific period and supported as an ethical or philosophical dimension like a violation of discretion. This age of reasoning emerged in the 1650s to 1800s as exemplified by a priority for human rights leading to its study in the humanities, philosophy, mathematics and other sciences. The economic revolution, however, brought more conventional thought and reform-oriented criminology like classical school seen within the Progressive Era. Cesare Lombroso (1835–1909), the psychiatrist who is recognised as the father of forensic science is indelibly marked as the pioneer for the origin and course of theories about biological criminal activities [3]. Within the late 1990s and early 2000s, a revival of biosocial ideas as criminal acts have been experienced and viewed as a results of biological factors interacting with each other within the past and present environments [4,5]. Additionally, the interaction between genes and therefore the environment relatively affects anti-social behaviour and shared environmental factors like family crime, poverty, poor parenting which have a significant effect on an individual committing a criminal offence, particularly a non-aggressive offence [5]. But, early health risk factors like mild physical anomalies, exposure to nicotine or alcohol and complications at birth combined with environmental risk factors are also seen to be predisposed to crime [5,6].

* Corresponding author.
E-mail address: nicholas@aims.edu.gh (N.K.-D. Ohene Opoku).

https://doi.org/10.1016/j.chaos.2021.110801
0960-0779/

Consistent with routine activity (RA) theory, different types of holiday periods have their associated dominant crime [7]. It has been found that major holidays were highly related to expressive crimes. As an example, sexual abuse increases during Valentine's Day and theft cases rise around the Christmas Season as well as entertainment events. This is confirmed by the RA which suggested that major holidays or festive periods were more likely to influence and alter the standard daily activities of people as well as bringing family and friends together in an environment. These gatherings are mostly related to a high level of alcohol consumption and could increase the propensity of one committing a criminal offence. Consistent with National Insurance Crime Bureau (NICB) report in 2018, the subsequent number of daily theft cases that were recorded within the various holidays in the USA are as follows; New Year Day (2571), President Day (2380), Halloween (2275), Labour Day (2235), Memorial Day (2167), Valentine Day (2001) and Christmas Day (1912). Crime does not only affect economic productivity but it has a diverse effect on the nation as well. Controlling crime considers the very fact that crime may happen at any event or festive period. Hence, it is necessary to reduce crime by keeping it at a minimal level to avoid it escalating [8].

In the study and analysis of crime, some mathematical models have been used. A population-based epidemiological approach was utilized in modelling crime, where it was believed that crime would spread through social interaction [9]. Stanford researchers, Berestycki et al. [10], used a reaction-diffusion-advection model to explain and reduce the spread of crime waves from crime hotspots. Criminal behaviours and violence can also be treated as a socially communicable disease using epidemiological-borrowed concepts [9,11]. These studies recognized the propensity of violent acts to cluster, spread and mutate from one area to another and based on using existing mathematical epidemiology techniques they have been able to treat the spread of violence in a population [11].

A similar model for gang growth in a population was developed by dividing the population into four distinct categories based on gang affiliation and risk factors for gang membership in Sooknanan et al. [12]. Their model analyzed the effect of varying crime-fighting approaches by adjusting parameter values like imprisonment and recurrence levels. They equally establish bifurcation points that resulted in gang members disappearing from the community. Furthermore, Saldana et al. [13] presented a nonlinear model of urban burglar dynamics which takes into consideration the police deterrent effect. The predator-prey approach was used where houses served as prey and burglars as predators. Their model focuses on the timing of criminal incidents, the spatial distribution of burglaries and population patterns supported by age. The burglars' tendency to commit a criminal offence and the pace at which houses were burgled were analyzed using repeat patterns of victimization. The asymptotic essence of the model together with the presence of global equilibrium stability was obtained through a continuous re-scaling of variable time. However, it is assumed in their model that the nature and structure of the houses might be an element that influences the speed at which these houses are going to be burgled but this factor was not considered.

Also, Raimundo et al. [14] examined how criminal behaviours are often contained and treated using Partially Contagious Criminality Model (PCCM). Their model include social, economic, personal and peer-pressure that determined the likelihood that a susceptible person with criminal propensity will engage in a criminal career. Their model further include individuals who have a criminal propensity but have never been imprisoned, individuals who are vulnerable to criminal activity and once jailed and those who are jailed several times. The jailed population were; first-timers and multiple times jailed at a given time. Lastly, those who have the propensity to commit a criminal offence but abstained from it either by their efforts or early interventions and those who are jailed for the primary time or several times but are relieved of criminal behaviours were captured. In their findings, there was an endemic equilibrium of criminality even when the essential reproduction number for contagion is below unity.

From the models reviewed, it is observed that building a crime model involves a multidisciplinary approach to bridging the gap between the physical and social sciences. The perfect sort of labour division in quantitative social science would be one where the sociologist would formulate a theory, mathematician translates it into a mathematical model, statistician vides the models to exhibit the parameters and computer scientist performs numerical simulations [15]. Thus, in this paper we develop a model that seek to control crime during entertainment events or festive periods using mathematical techniques.

The rest of the paper is organised as follows. In Section 2, the model is formulated and the analysis of the model is presented in Section 3. In Section 4, the optimal control equations are developed and analysed together with the numerical simulations. The section further accounts for the associated cost of controlling the crime. Concluding remarks are described in Section 5.

2. Model formulation

In this section, the formulation of the crime transmission model is made and it classifies the total population of individual in the society T into four compartments depending on an individual's criminal (infection) status. Compartment labeled S, represent the susceptible individuals in the population, N denotes individuals with the intention to commit crime, I account for infected (active criminals) individuals and R shows those who have recovered from criminal activities. Hence the total population is given by,

$$T = S(t) + N(t) + I(t) + R(t).$$

In the model, the influx of individuals are recruited into the event at a rate given by μT. The parameter ϕ is the rate at which individuals who are recruited into the susceptible population have the intention to commit crime. Therefore, individuals with the intention to commit crime move from the susceptible compartment to compartment N. The susceptible and individuals with the intention to commit crime become criminal at the transmission rate β and $\beta(1-\theta)$, where θ is the modification of the behaviour of those having the intention to commit crime. Thus, if the value for $\theta = 1$, then it implies that there is a positive modification of their behaviour. But if $\theta = 0$, then there is a full transmission from N compartment to the I compartment. We further make the assumption that individuals who are criminals recover through the desistance rate given by ξ. The desistance rate is a function of the duration $(d > 0)$ of the programme and the number of infected individuals (I) at time t. The desistance function $\xi(d, I)$, is given by

$$\xi(d, I) = \left[\xi_0 + \left(\xi_1 - \xi_0\right)\frac{d}{d+I}\right]I,$$

where ξ_0 represents the minimum per capital desistance rate and ξ_1 is the maximum desistance rate such that $\xi_1 > \xi_0 > 0$. The function $\xi(d, I)$ investigates the impact of the duration of the programme on the spread and control of crime. The constant population under consideration in the model is further assumed to have a natural exit rate given by μ. A schematic representation of the above model is shown in Fig. 1 and it associated state variables and parameters are described in Table 1.

3. Model dynamics

The following set of non-linear ordinary differential equations are obtained from the assumptions and flow diagram described in

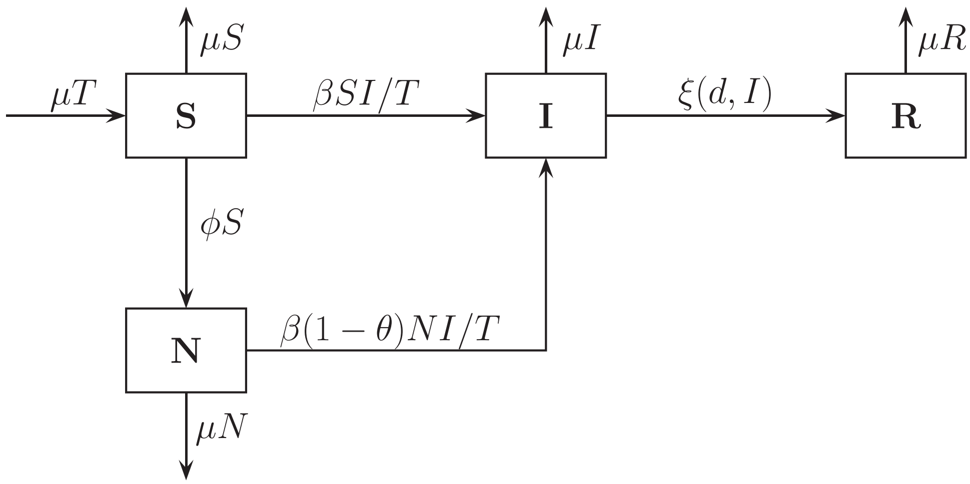


Fig. 1. Schematic figure of crime infection model.

Table 1
Description of parameters in the model.

State variables	Description
S	susceptible individuals
N	individuals with intention to commit crime
I	active criminals
R	individuals stopping crime on their own
T	total number of individuals in the society
Parameters	Description
β	transmission rate
μ	rate at which individual exit from an event
ϕ	rate at which individual with the intention to commit crime
θ	efficacy rate
d	duration or length of an event
ξ	desistance rate
ξ_1	maximum desistance rate
ξ_0	minimum per capita desistance rate

Section 2 as;

$$\begin{cases} \dot{S} = \mu T - \mu S - \frac{\beta SI}{T} - \phi S, \\ \dot{N} = \phi S - \frac{\beta NI}{T}(1-\theta) - \mu N, \\ \dot{I} = \frac{\beta SI}{T} + \frac{\beta NI}{T}(1-\theta) - \mu I - \xi(d, I), \\ \dot{R} = \xi(d, I) - \mu R, \end{cases} \tag{1}$$

where the dots on each variables (S, N, I, R) denote the first derivative with respect to time t. The initial conditions are such that $S(0) > 0,\ N(0) \geq 0,\ I(0) \geq 0,\ R(0) \geq 0$. From model (1), we observe that the dimension of human population is not the same as the length of the programme. As a result, to make the system dimensionless, we apply the dimensionless technique and make the following substitutions such that; $S = sT,\ N = nT,\ I = iT,\ R = rT$ and $d = \hat{d}T$ with $s + n + i + r = 1$. The new system of equations becomes;

$$\begin{cases} \dot{s} = \mu - \mu s - \beta si - \phi s, \\ \dot{n} = \phi s - \beta(1-\theta)ni - \mu n, \\ \dot{i} = \beta si + \beta(1-\theta)ni - \mu i - \left[\xi_0 + \left(\xi_1 - \xi_0\right)\frac{\hat{d}}{\hat{d}+i}\right]i, \\ \dot{r} = \left[\xi_0 + \left(\xi_1 - \xi_0\right)\frac{\hat{d}}{\hat{d}+i}\right]i - \mu r. \end{cases} \tag{2}$$

3.1. Well-posedness of the model

In this section, we illustrate that model (2) is sociologically and mathematically well defined in the positive invariant domain given as

$$\Omega = \{(s + n + i + r) \in \mathbb{R}^4_+ \leq 1\}.$$

Lemma 1 (Positively invariant set)**.** *The region Ω with initial conditions $(s(0) > 0,\ n(0) \geq 0,\ i(0) \geq 0,\ r(0) \geq 0)$ for all $t_0 > 0$ is bounded, positively invariant and attracting with respect to the model (2).*

Proof. We define

$$t_0 = \sup\{t > 0 : s(t) > 0 \quad \text{and} \quad (n(t),\ i(t),\ r(t)) \geq 0\}.$$

Thus,

$$s(t) > 0 \quad \text{and} \quad n(t),\ i(t),\ r(t) \geq 0 \quad \forall t \in [0, t_0).$$

Considering the first equation in the model (2) we have:

$$\frac{ds}{dt} = \mu - \mu s - \beta si - \phi s, \\ \geq -(\mu + \beta i + \phi)s, \quad t \in [0, t_0).$$

By separation of variables, we have

$$\frac{ds}{s} \geq -(\mu + \beta i + \phi)dt.$$

Now integrating both sides from 0 to t_0 yields;

$$\int_0^{t_0} \frac{ds}{s} \geq \int_0^{t_0} -(\mu + \beta i + \phi)dt.$$

Hence we have

$$s(t_0) \geq s(0)e^{-\left(\mu t_0 + \phi t_0 + \int_0^{t_0}(\beta i dt)\right)} > 0.$$

From the second equation in model (2), we can write;

$$\frac{dn}{dt} \geq (-\beta(1-\theta)i - \mu)n \quad t \in [0, t_0).$$

By separation of variables we have;

$$\frac{dn}{n} \geq [-\beta(1-\theta)i - \mu]dt.$$

Integrating both sides from 0 to t_0 gives;

$$\int_0^{t_0} \frac{dn}{n} \geq \int_0^{t_0} \left(-\beta(1-\theta)i - \mu\right)dt,$$

$$\ln(n(t_0)) \geq -\mu t_0 - \int_0^{t_0} (\beta(1-\theta)i)dt.$$

Therefore,

$$n(t_0) \geq n(0)e^{\left(-\mu t_0 - \int_0^{t_0}(\beta(1-\theta)i)dt\right)} \geq 0.$$

In a similar manner, the following solutions are respectively obtained for the third and fourth equation in model (2),

$$i(t_0) \geq i(0)e^{-(\mu+(\xi_1-\xi_0)\hat{d}/(\hat{d}+i))t_0} \geq 0$$

and

$$r(t_0) \geq r(0)e^{-\mu t_0} \geq 0.$$

Therefore, $s(t) > 0,\ n(t) \geq 0,\ i(t) \geq 0,$ and $r(t) \geq 0$ for all time $t_0 > 0$. □

3.2. Steady states and basic reproduction number

We obtain two non-negative equilibrium points which are crime-free equilibrium (CFE) and crime endemic equilibrium (CEE). We obtain the crime-free equilibrium points as;

$$CFE = (s^0, n^0, i^0, r^0) = \left(\frac{\mu}{\mu+\phi}, \frac{\phi}{\mu+\phi}, 0, 0\right).$$

3.2.1. Basic reproduction number

The basic reproduction number, R_0, of model (2) is defined as a threshold parameter that measures the average number of new criminals produced by the relapse and interaction of the criminal population with the susceptible population [16]. Using the concept in [17], the basic reproduction number is obtained as

$$R_0 = \frac{\beta(\mu+\phi(1-\theta))}{(\xi_1+\mu)(\mu+\phi)}.$$

3.3. Local stability of CFE

In this section, we investigate the stability analysis of the crime-free equilibrium by proving the following theorem.

Theorem 1. *The crime-free equilibrium, CFE, is locally asymptotically stable if $R_0 < 1$, otherwise model (2) is unstable.*

Proof. The Jacobian matrix for model (2) at CFE is given by

$$J(CFE) = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \tag{3}$$

where

$$A = \begin{pmatrix} -\mu-\phi & 0 \\ \phi & -\mu \end{pmatrix}, \quad B = \begin{pmatrix} -\frac{\beta\mu}{(\mu+\phi)} & 0 \\ -\frac{\beta\phi(1-\theta)}{(\mu+\phi)} & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad D = \begin{pmatrix} \rho & 0 \\ -\xi_1 & -\mu \end{pmatrix},$$

and

$$\rho = \left(\tfrac{\beta\mu}{(\mu+\phi)} - \mu - \xi_1 + \tfrac{\beta\phi(1-\theta)}{(\mu+\phi)}\right).$$

The eigenvalues for Eq. (3) are given by the diagonal entries of the lower triangular of matrix A and D. We therefore deduce that the four eigenvalues are:

- The first and second eigenvalues are given by $-\mu \leq 0$.
- The third eigenvalue is given by $-(\mu+\phi) \leq 0$.
- The fourth eigenvalue is given by ρ.

The condition for the fourth eigenvalue to be negative is such that

$$\rho = \frac{\beta(\mu+\phi(1-\theta))}{\mu+\phi} - (\mu+\xi_1) < 0,$$

which simplifies to

$$R_0 = \frac{\beta(\mu+\phi(1-\theta))}{(\mu+\phi)(\xi_1+\mu)} < 1.$$

Therefore, the crime-free equilibrium steady state is locally-asymptotically stable for $R_0 < 1$. □

3.4. Crime endemic equilibrium (CEE)

To find the endemic equilibrium we equate the derivatives of model (2) to zero such that $(s^* \neq 0,\ n^* \neq 0,\ i^* \neq 0,\ r^* \neq 0)$ to obtain

$$0 = \mu - \mu s^* - \beta s^* i^* - \phi s^*, \tag{4}$$

$$0 = \phi s^* - \beta(1-\theta)n^* i^* - \mu n^*, \tag{5}$$

$$0 = \beta s^* i^* + \beta(1-\theta)n^* i^* - \mu i^* - \xi(\hat{d}, i^*), \tag{6}$$

$$0 = \xi(\hat{d}, i^*) - \mu r^*, \tag{7}$$

where $\eta(\hat{d}, i^*) = \left[\xi_0 + (\xi_1 - \xi_0)\frac{\hat{d}}{\hat{d}+i^*}\right]$ and $\xi(\hat{d}, i^*) = \eta(\hat{d}, i^*)i^*$. The endemic equilibrium point for the state variables obtain as

$$s(i^*) = \frac{\mu}{\beta i^* + \mu + \phi}, \tag{8}$$

$$n(i^*) = \frac{\mu\phi}{(\beta i^*(1-\theta)+\mu)(\beta i^* + \mu + \phi)}, \tag{9}$$

$$r(i^*) = \frac{i^*(\xi_0 i^* + \xi_1 \hat{d})}{(\hat{d}+i^*)\mu}$$

$$0 = [\beta s + \beta(1-\theta)n - \eta(\hat{d}, i^*) - \mu]i^*. \tag{10}$$

From Eq. (10) we have,

$$0 = \beta s + \beta(1-\theta)n - \eta(\hat{d}, i^*) - \mu, \tag{11}$$

which is equal to

$$(\xi_1 - \xi_0)\frac{\hat{d}}{\hat{d}+i^*} = \beta s + \beta(1-\theta)n - \mu - \xi_0, \tag{12}$$

$$\hat{d}(\xi_1 - \xi_0) = (\hat{d}+i^*)[\beta s + \beta(1-\theta)n - \mu - \xi_0]. \tag{13}$$

From Eqs. (8) and (9) we let;

$$s(i^*) = \frac{\omega_1}{Y_1}, \quad n(i^*) = \frac{\omega_2}{Y_1 Y_2},$$

where $\omega_1 = \mu,\ \ Y_1 = \beta i^* + \mu + \phi,\ \ \omega_2 = \mu\phi,\ \ Y_2 = \beta(1-\theta)i^* + \mu$. Multiplying Eq. (13) by Y_1 and Y_2 gives

$$\hat{d}(\xi_1 - \xi_0)Y_1 Y_2 = (\hat{d}+i^*)[\beta s^* + \beta(1-\theta)n^* - \mu - \xi_0]Y_1 Y_2. \tag{14}$$

Moreover,

$$(\hat{d}+i^*)[\beta\mu Y_2 + \beta(1-\theta)\mu\phi] - (\hat{d}+i^*)(\mu+\xi_0)Y_1 Y_2 = \hat{d}(\xi_1 - \xi_0)Y_1 Y_2. \tag{15}$$

Eq. (15) is a polynomial of the form;

$$p_3(i^*) = a_3 i^{*3} + a_2 i^{*2} + a_1 i^* + a_0, \tag{16}$$

where;

$$a_0 = -[(\phi\theta - \mu - \phi)\beta + (\mu+\xi_1)(\mu+\phi)]\hat{d}\mu,$$

$$a_1 = -\hat{d}\mu(\theta-1)\beta^2 + [[(1+(\theta-2)\hat{d})\mu^2 + [((\phi+\xi_1)\theta - \phi - 2\xi_1]\hat{d} - \phi(\theta-1)]\mu, + \hat{d}\phi\xi_1(\theta-1))\beta - \mu(\mu+\xi_0)(\mu+\phi)],$$

$$a_2 = [((\hat{d}-1)\mu + \hat{d}\xi_1)(\theta-1)\beta + ((\theta-2)\mu + \phi(\theta-1))(\mu+\xi_0)]\beta,$$

$$a_3 = -\beta^2(1-\theta)(\mu+\xi_0).$$

Further simplification and algebraic manipulation, we have;

$$\begin{aligned}
a_0 &= (R_0-1)(\mu+\phi)(\mu+\xi_1)\hat{d}\mu,\\
a_1 &= \beta^2\hat{d}\mu(1-\theta) - \beta[(\hat{d}-1)\mu^2 + \hat{d}\xi_1\mu\\
&\quad + (1-\theta)\hat{d}\mu^2 + (1-\theta)[(\phi+\xi_1)\mu - \hat{d}\xi_1\phi]]\\
&\quad - \mu(\mu+\xi_0)(\mu+\phi).\\
a_2 &= -\beta(1-\theta)[\beta(\hat{d}-1)\mu + \beta\hat{d}\xi_1 + (\mu+\phi)(\mu+\xi_0)]\\
&\quad - \beta\mu(\mu+\xi_0),\\
a_3 &= -\beta^2(1-\theta)(\mu+\xi_0).
\end{aligned}$$

We immediately conclude that for $R_0 > 1$, the coefficients a_0 is positive while $a_3 < 0$ for all cases. The situation for a_1 and a_2 are not as simple.In fact while $a_1 < 0$ for R_0 slightly larger than unity, it turns to be become positive for $R_0 >> 1$. Finally, we can proof that the polynomial $p_3(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ satisfies that $p_3(0) = a_0 > 0$ and $p_3(1) = a_0 + a_1 + a_2 + a_3 < 0$. This follows from

$$\begin{aligned}
p_3(1) &= -(1-\theta)[\hat{d}\xi_1 + \xi_0]\beta^2 - [(1-\theta)(1+\hat{d})\mu^2\\
&\quad + (1+1-\theta)[\hat{d}\xi_1+\xi_0]\mu + \phi(1-\theta)[\hat{d}\xi_1+\xi_0]]\beta\\
&\quad - (\mu+\phi)[(1+\hat{d})\mu + \hat{d}\xi_1 + \xi_0]\mu.
\end{aligned}$$

It is obvious that $p_3(1) < 0$. Hence, we have shown that $p_3(t)$ has a sign change in the interval $0 < t < 1$. Thus, there exist a i^* with $0 < i^* < 1$ such that $p_3(i^*) = 0$.

3.5. Existence of a bifurcation

To show the existence of a bifurcation, we let the assumed bifurcation parameter be $\beta = \beta^*$ at $R_0 = 1$. Thus, the bifurcation parameter, β^*, is obtained as

$$\frac{\beta^*(\mu+\phi(1-\theta))}{(\mu+\phi)(\mu+\xi_1)} = 1,$$

$$\beta^* = \frac{(\mu+\phi)(\mu+\xi_1)}{(\mu+\phi(1-\theta))}. \tag{17}$$

We establish conditions for the existence of backward bifurcation following Theorem 4.1 proven in Brauer [18]. Let $\mathbf{x} = (s,n,i,r)^T$ so that model (2) can be restructured as $\frac{d\mathbf{x}}{dt} = \mathbf{f}$, where $\mathbf{f} = (f_1, f_2, f_3, f_4)^T$. The crime model (2) becomes

$$\begin{cases}
\frac{ds}{dt} = \mu - \mu s - \beta^* si - \phi s = f_1,\\
\frac{dn}{dt} = \phi s - \beta^*(1-\theta)ni - \mu n = f_2,\\
\frac{di}{dt} = \beta^* si + \beta^*(1-\theta)ni - \mu i - \left[\xi_0 + (\xi_1-\xi_0)\frac{\hat{d}}{\hat{d}+i}\right]i = f_3,\\
\frac{dr}{dt} = \left[\xi_0 + (\xi_1-\xi_0)\frac{\hat{d}}{\hat{d}+i}\right]i - \mu r = f_4.
\end{cases} \tag{18}$$

The Jacobian matrix of Eq. (18) at CFE is given as

$$J(CFE) = \begin{pmatrix}
-\mu-\phi & 0 & -\beta^* s & 0\\
\phi & -\mu & -\beta^*(1-\theta)n & 0\\
0 & 0 & \beta^* s - \mu - \xi_1 + \beta^*(1-\theta)n & 0\\
0 & 0 & \xi_1 & -\mu
\end{pmatrix}. \tag{19}$$

Eq. (19) has a a simple eigenvalue implying that the center manifold theory can be used to analyze the dynamics of model (2) at $\beta = \beta^*$. To proceed, we find the right eigenvector of Eq. (19) as

$$\begin{bmatrix}
-\mu-\phi & 0 & -\beta^* s & 0\\
\phi & -\mu & -\beta^*(1-\theta)n & 0\\
0 & 0 & \beta^* s - \mu - \xi_1 + \beta^*(1-\theta)n & 0\\
0 & 0 & \xi_1 & -\mu
\end{bmatrix}
\begin{bmatrix} w_1\\ w_2\\ w_3\\ w_4\end{bmatrix}
= \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \tag{20}$$

where $\mathbf{w} = (w_1, w_2, w_3, w_4)^T$ is the right eigenvector. Solving for (w_1, w_2, w_3, w_4), we obtained the following results:

$$\begin{aligned}
w_1 &= -\frac{\beta^*\mu}{(\mu+\phi)^2},\\
w_2 &= -\frac{(\mu+\phi)^2\beta^*(1-\theta)n + \beta^*\mu\phi}{(\mu+\phi)^2\mu}\\
w_3 &= 1,\\
w_4 &= \frac{\xi_1}{\mu}.
\end{aligned}$$

The corresponding left eigenvector is equally obtained from the transposed Jacobian matrix (19) associated with the zero eigenvalue which is given $\mathbf{v} = (v_1, v_2, v_3, v_4)^T$. Solving for (v_1, v_2, v_3, v_4) yields

$$\mathbf{v} = \big(0, 0, 1, 0\big).$$

Since v_1, v_2 and v_4 vanish the second derivatives of f_1, f_2 and f_4 do not influence the local behavior close to the bifurcation point. The local dynamics of model (2) around the equilibrium point is completely determined by the signs of $\mathbf{a}$ and $\mathbf{b}$ as established in Castillo-Chavez and Song [19], Opoku et al. [20]. Computing for $\mathbf{a}$, the corresponding non-zero partial derivatives of $\mathbf{f}$ at the crime-free equilibrium is given as

$$\frac{\partial^2 f_3}{\partial s \partial i} = \beta^*, \quad \frac{\partial^2 f_3}{\partial n \partial i} = \beta^*(1-\theta), \quad \frac{\partial^2 f_3}{\partial i^2} = \frac{2(\xi_1-\xi_0)}{\hat{d}}.$$

Thus,

$$\begin{aligned}
\mathbf{a} &= 2v_3 w_1 w_3 \frac{\partial^2 f_3}{\partial s\partial i} + 2v_3 w_2 w_3 \frac{\partial^2 f_3}{\partial n \partial i} + v_3 w_3^2 \frac{\partial^2 f_3}{\partial i^2},\\
&= v_3 w_3\left(2\beta^* w_1 + 2w_2\beta^*(1-\theta) + w_3\frac{2(\xi_1-\xi_0)}{\hat{d}}\right),\\
&= E + \frac{2(\xi_1-\xi_0)}{\hat{d}},
\end{aligned}$$

where

$$E = -\frac{2\beta^{*2}[\mu^2 + (1-\theta)^2(\mu+\phi)\phi + (1-\theta)\mu\phi]}{\mu(\mu+\phi)^2} < 0.$$

Therefore we have two conditions

- $\mathbf{a} < 0$, if $\left[E + \frac{2(\xi_1-\xi_0)}{\hat{d}} < 0\right]$,
- $\mathbf{a} > 0$, if $\left[E + \frac{2(\xi_1-\xi_0)}{\hat{d}} > 0\right]$.

Computing for $\mathbf{b}$, the corresponding non-zero partial derivative of $\mathbf{f}$ at the crime-free equilibrium is given as

$$\frac{\partial^2 f_3}{\partial i \partial \beta^*} = s + (1-\theta)n, \quad \frac{\partial^2 f_3}{\partial n \partial \beta^*} = (1-\theta)i^* = 0,$$
$$\frac{\partial^2 f_3}{\partial s \partial \beta^*} = i = 0.$$

$$\begin{aligned}
\mathbf{b} &= v_3 w_3 (s + (1-\theta)n),\\
&= \frac{\mu + (1-\theta)\phi}{(\mu+\phi)}.
\end{aligned}$$

Thus, $\mathbf{b} > 0$. Hence, we have the following theorem;

Theorem 2. *There exist a backward bifurcation at $R_0 = 1$ if $\mathbf{a} > 0$ and $\mathbf{b} > 0$.*

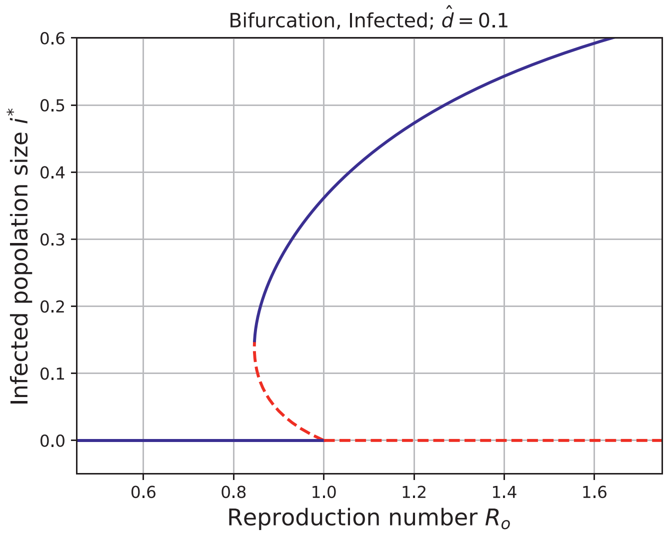


Fig. 2. Backward bifurcation.

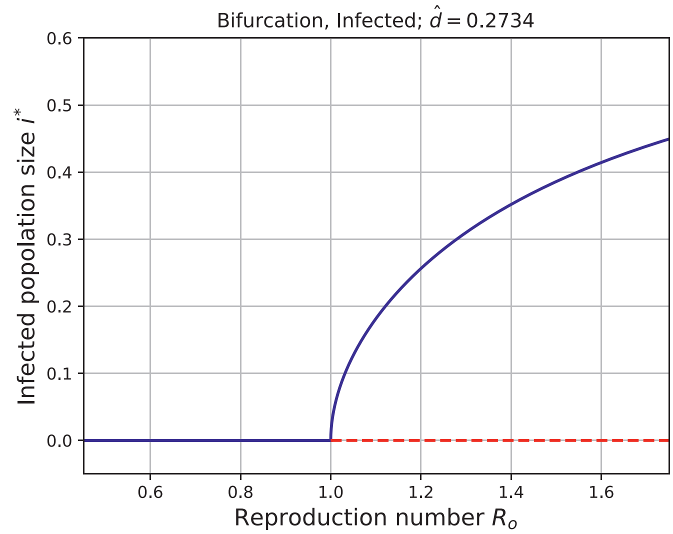


Fig. 3. Forward bifurcation.

Table 2
Model parameters and their baseline values.

Parameter	Base-line value	Units	Reference
β :	0.58	hour^{-1}	[24]
μ :	0.115	hour^{-1}	Assumed
ϕ:	0.1265	hour^{-1}	Assumed
θ:	0.44	hour^{-1}	[24]
$\hat{d}$:	5.0	–	Assumed
ξ_1:	0.14	hour^{-1}	Assumed
ξ_0:	0.0015	hour^{-1}	Assumed

We observe that a backward bifurcation occurs at $R_0 = 1$. Hence, the existence of a backward bifurcation is illustrated by the use of a numerical approach (see Fig. 2) by creating the bifurcation curve using the values in Table 2 and fixing $\hat{d} = 0.1$.

From the epidemiological point of view, when a model exhibits the phenomenon of backward bifurcation [20], it means that the stable endemic equilibrium may also exist when $R_0 < 1$. When this happens, it means the reproduction number is no longer sufficient to guarantee crime elimination. Hence, the crime reproduction number must be reduced under a smaller threshold in order to avoid multiple endemic states and get crime elimination. The stability region of the model is related to the value of the parameter $\hat{d}$. Thus, to obtain a forward bifurcation we set $\hat{d} = 0.2734$ to obtain Fig. 3.

Based on Fig. 3, it implies that crime can be controlled depending on the length or duration of the event. However, the occurrence of a bistability phenomenon when R_0 is less than 1 as shown in Fig. 2 indicate the need for the implementation of other control measures irrespective of the length or duration of the programme during an event. As a result, the implementation of other control measures is introduced in the next section to help achieve crime elimination.

4. Optimal control problem

In this section, we introduce three control functions $c_1(t), c_2(t)$ and $c_3(t)$ into model (2) to obtain a modified model given as

$$\left\{\begin{aligned} \dot{s} &= \mu - (1-c_1)\beta si - \phi s - \mu s, \\ \dot{n} &= \phi s - (1-c_2)\beta(1-\theta)ni - \mu n, \\ \dot{i} &= (1-c_1)\beta si + (1-c_2)\beta(1-\theta)ni \\ &\quad -\left[\xi_0 + (\xi_1 - \xi_0)\frac{\hat{d}}{\hat{d}+i}\right]i - (1+c_3)\mu i, \\ \dot{r} &= \left[\xi_0 + (\xi_1 - \xi_0)\frac{\hat{d}}{\hat{d}+i}\right]i - \mu r. \end{aligned}\right\} \tag{21}$$

The associated control reproduction number, R_0^c, of Eq. (21) is obtained as

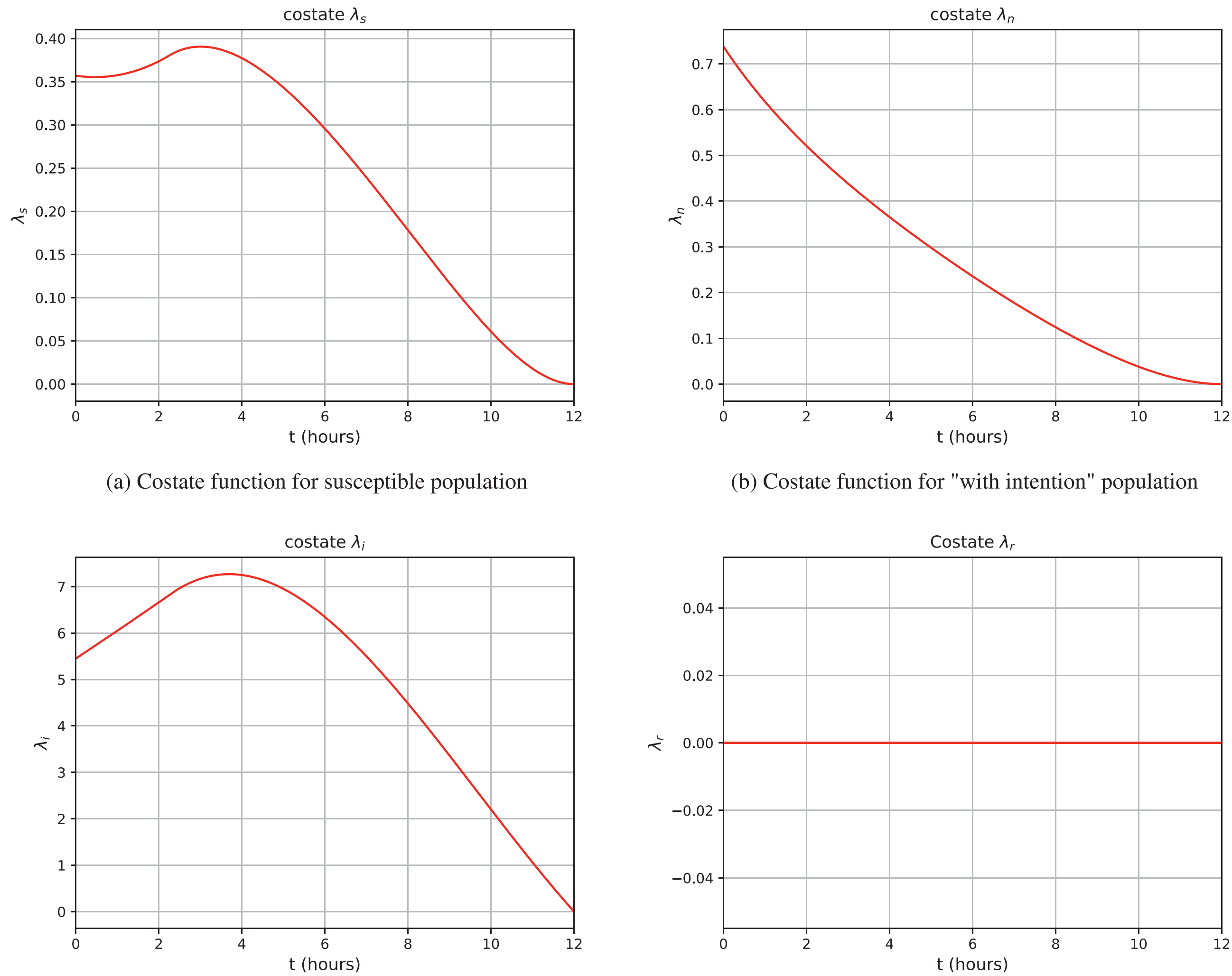
$$R_0^c = \frac{\beta[\mu(1-c_1) + \phi(1-c_2)(1-\theta)]}{[\xi_1 + \mu(1+c_3)](\mu+\phi)}. \tag{22}$$

The control function c_1 measures the level of success achieved in educating people about possible occurrence of criminal acts during events. This will reduce the intensity of criminals on influencing susceptible individuals to become criminals. The function c_2 represents detaining people who are found to be engaging in any criminal activity or behaviour and c_3 represents the effort of sacking anyone with criminal behaviour from the event premises. Sacking those people will deter others from showing behaviour that will breed any criminal activities. It is crucial to know that if $c_1 = c_2 = c_3 = 1$, then it suggests that the mentioned interventions are 100% effective on attaining the desired goal. But if $c_1 = c_2 = c_3 = 0$, then it implies that the various interventions are not effective. Hence, we set up an objective functional, J, consisting $M_3 > 0$ which balances the coefficients of the infected state variable (i). Considering the following objective functional over a set of feasible (c_1, c_2, c_3) interventions between the period $[0, t_f]$ gives

$$J(c_1, c_2, c_3) = \int_0^{t_f}\left[M_3 i + \frac{W_1}{2}c_1^2 + \frac{W_2}{2}c_2^2 + \frac{W_3}{2}c_3^2\right]dt, \tag{23}$$

where W_1, W_2, W_3 are weight constants on the controls. By using Pontryagin's Maximum Principle (PMP) [21], the necessary conditions that the optimal controls must satisfy are obtained. Therefore, model (21) is transformed into an equivalent problem, namely the problem of minimising the Hamiltonian H with respect to the given controls. Thus,

$$\begin{aligned} H = &\left[M_3 i + \frac{W_1}{2}c_1^2 + \frac{W_2}{2}c_2^2 + \frac{W_3}{2}c_3^2\right] \\ &+ \lambda_s[\mu - (1-c_1)\beta si - \phi s - \mu s] \\ &+ \lambda_n[\phi s - (1-c_2)\beta(1-\theta)ni - \mu n] \\ &+ \lambda_i\Big[(1-c_1)\beta si + (1-c_2)\beta(1-\theta)ni \\ &- \left(\xi_0 + (\xi_1 - \xi_0)\frac{\hat{d}}{\hat{d}+i}\right)i - (1+c_3)\mu i\Big] \\ &+ \lambda_r\left(\left[\xi_0 + (\xi_1 - \xi_0)\frac{\hat{d}}{\hat{d}+i}\right]i - \mu r\right). \end{aligned} \tag{24}$$



(a) Costate function for susceptible population

(b) Costate function for "with intention" population

(c) Costate function for infected population

(d) Costate function for recovery population

Fig. 4. Simulation of model (21) showing the respective costate variables.

The parameters $\lambda_s,\ \lambda_n,\ \lambda_i$ and λ_r in Eq. (24) are the co - state variables associated with the state variables $(s,\ n,\ i,\ r)$. Given the following optimal controls, $c_1^*,\ c_2^*,\ c_3^*$, and state variables, there exists an adjoint functions satisfying:

$$\begin{cases} \dot{\lambda_s} = (\lambda_s - \lambda_i)[(1-c_1)\beta i] + \lambda_s(\phi+\mu) - \lambda_n\phi, \\ \dot{\lambda_n} = (\lambda_n - \lambda_i)[(1-c_2)(1-\theta)\beta i] + \lambda_n\mu, \\ \dot{\lambda_i} = -M_3 + (\lambda_s - \lambda_i)\big[(1-c_1)\beta s\big] \\ \qquad +(\lambda_n - \lambda_i)\big[(1-c_2)\beta n(1-\theta)\big] \\ \qquad +(\lambda_i - \lambda_r)\Big[\xi_0 + (\xi_1 - \xi_0)\frac{\hat{d}^2}{(\hat{d}+i)^2}\Big] + \lambda_i(1+c_3)\mu, \\ \dot{\lambda_r} = \lambda_r\mu. \end{cases} \tag{25}$$

The following relations were used to derive the adjoint Eq. (25):

$$\frac{d\lambda_s}{dt} = -\frac{\partial H}{\partial s}, \quad \frac{d\lambda_n}{dt} = -\frac{\partial H}{\partial n}, \quad \frac{d\lambda_i}{dt} = -\frac{\partial H}{\partial i}, \quad \frac{d\lambda_r}{dt} = -\frac{\partial H}{\partial r}.$$

The relevant transversality conditions hold; $\lambda_s(t_f) = 0,\ \lambda_n(t_f) = 0,\ \lambda_i(t_f) = 0$ and $\lambda_r(t_f) = 0$. Furthermore, using $\frac{\partial H}{\partial c_i}$, we obtain,

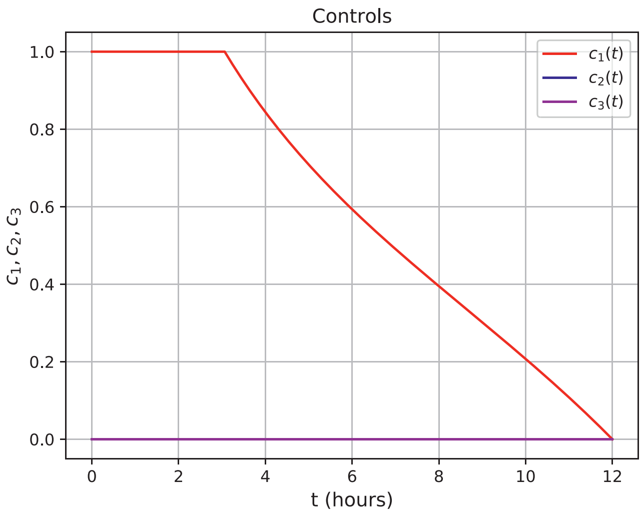
$$\begin{cases} \frac{\partial H}{\partial c_1} = W_1c_1 + (\lambda_s - \lambda_i)\beta si = 0, \\ \frac{\partial H}{\partial c_2} = W_2c_2 - (\lambda_i - \lambda_n)[\beta(1-\theta)ni] = 0, \\ \frac{\partial H}{\partial c_3} = W_3c_3 - \lambda_i\mu i = 0. \end{cases} \tag{26}$$

Since the optimal controls exhibit the characteristics for c_i^* from $\frac{\partial H}{\partial c_i}$ whenever $0 < c_i < 1$, taking the bounds into consideration, we have

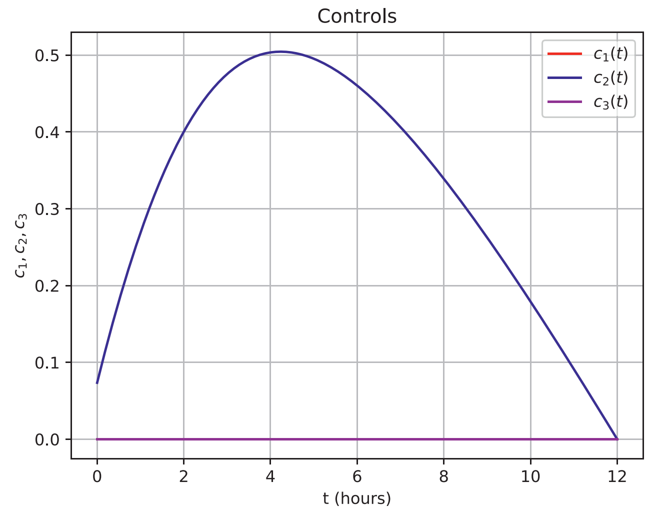
$$c_1^*(t) = \min\left\{\max\left\{0, \frac{(\lambda_i - \lambda_s)\beta si}{W_1}\right\}, 1\right\},$$
$$c_2^*(t) = \min\left\{\max\left\{0, \frac{(\lambda_i - \lambda_n)[(\beta(1-\theta)ni}{W_2}\right\}, 1\right\},$$
$$c_3^*(t) = \min\left\{\max\left\{0, \frac{\lambda_i\mu i}{W_3}\right\}, 1\right\}.$$

4.1. Numerical results

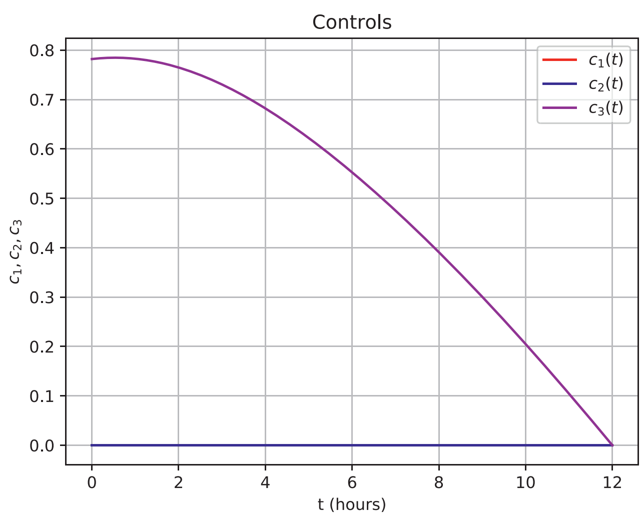
In this section, we simulate model (21) and the adjoint system of differential equations with the control characterisation from the optimal conditions. The optimality systems are solved numerically



(a) Control profile for $c_1 \neq 0$ and $c_2 = c_3 = 0$.



(b) Control profile for $c_2 \neq 0$ and $c_1 = c_3 = 0$.



(c) Control profile for $c_3 \neq 0$ and $c_1 = c_2 = 0$.

Fig. 5. Control profiles for implementing a single control.

using the forward-backward sweep method (FBSM) of which details concerning it application can be found in [22,23]. For $M_3 > 0$ we used the parameter values in Table 2 and the following values for the weights and state variables to obtain the numerical results.

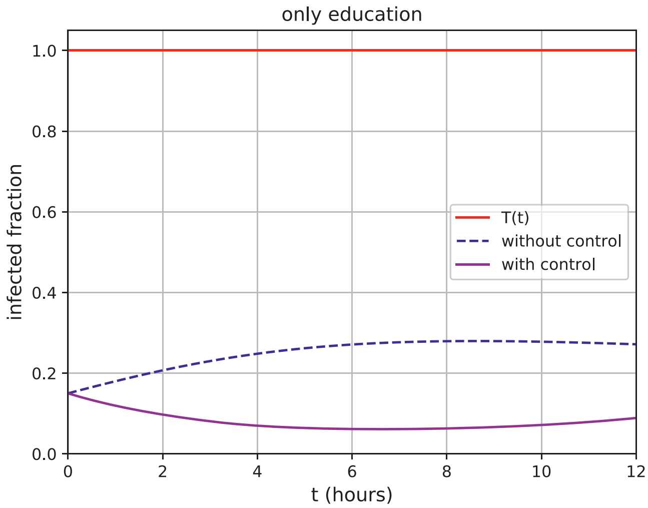
$$W_1 = 0.2,\ W_2 = 0.2,\ W_3 = 0.2,\ M_3 = 1,\ s = 0.77,\ n = 0.03,\ i = 0.15,\ r = 0.05.$$

We begin by investigating some fundamental properties of Eq. (24). With our steady states, and since we have finite horizon version, we imposed no conditions on the terminal value of $s(t_f),\ n(t_f),\ i(t_f),\ r(t_f)$. This implies that the relevant transversality conditions $\lambda_s(t_f) = 0,\ \lambda_n(t_f) = 0,\ \lambda_i(t_f) = 0,\ \lambda_r(t_f) = 0$ for each co-state variable holds. The path in the co-state of the state variables satisfies the necessary conditions for optimality as shown in Fig. 4. The path corresponding to each co-state variable decreases monotonically reaching $\lambda_s(t_f) = 0,\ \lambda_n(t_f) = 0,\ \lambda_i(t_f) = 0,\ \lambda_r(t_f) = 0$ respectively at time $(t_f) = 12$ h. This is clearly reflected in the respective co-state variables of which our current-value Hamiltonian is quadratic in crime incidence. This leads to a shadow cost of crime associated with $s(t_f),\ n(t_f)$, and $i(t_f)$ except for the co-state function of the recovery population, $r(t_f)$, since there is no control on it. These control interventions were designed such that they minimise the population of criminals during an event.

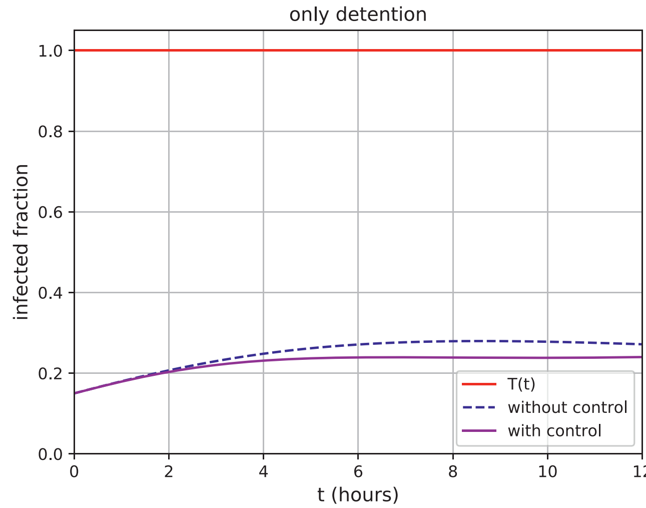
4.1.1. Strategy I: application of a single control

In this subsection, we begin by investigating the effect of applying a single control and ascertain it effectiveness in reducing crime during an event. The control profiles for each single control is shown in Fig. 5. It is prudent to note that for Fig. 5a, the line graph of c_2 is embedded in that of c_3. In a similar manner, the line graph of c_1 in Fig. 5b is also embedded in c_3. Moreover, in Fig. 5c, the line graph of c_1 is embedded in c_2.

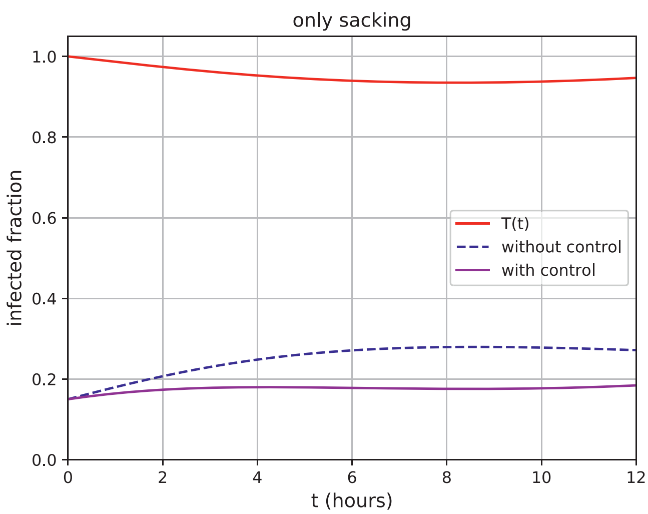
Now, we check the respective single controls on the infected population to check their effect in reducing crime during events. Fig. 6a represents the effectiveness of implementing only educa-



(a) Effect of education only in control crime.



(b) Effect of detention only in control crime.



(c) Effect of sacking only in control crime.

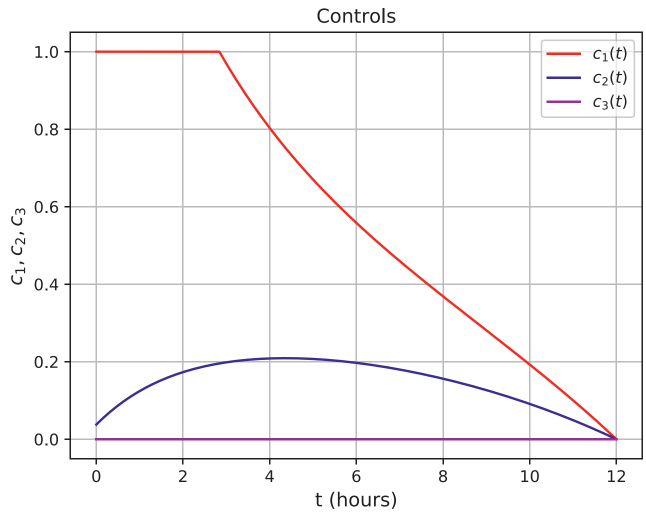
Fig. 6. Effect of applying a single control intervention on model (21).

tion. It can be observed that when education is applied, the total number of people in the infected compartment reduces and since education is such that it does not take people out of the event premises we observed the total population ($T(t) = 1$) to be constant. Alternatively, we observed that the infected population increases indicated by the blue dotted line when such no intervention is applied. Fig. 6b denotes the effectiveness of applying only detention as a control measure. It can be seen that the infected population equally reduces when the control is applied. However, the total population remain constant indicating that individuals who exhibit criminal activity are detained for a particular period of time at the event premises. Moreover, the observed infected population grows exponentially if such intervention is not put in place. Fig. 6c also shows the effectiveness of applying the control measure by sacking individuals with observed criminal behavior from an event. When we apply only that control the individuals in the infected population reduces and the infected population increases without applying the control. Because individuals with criminal behaviours are sacked from the event, it reduces the total population, $T(t)$, starting from the time the control was applied.

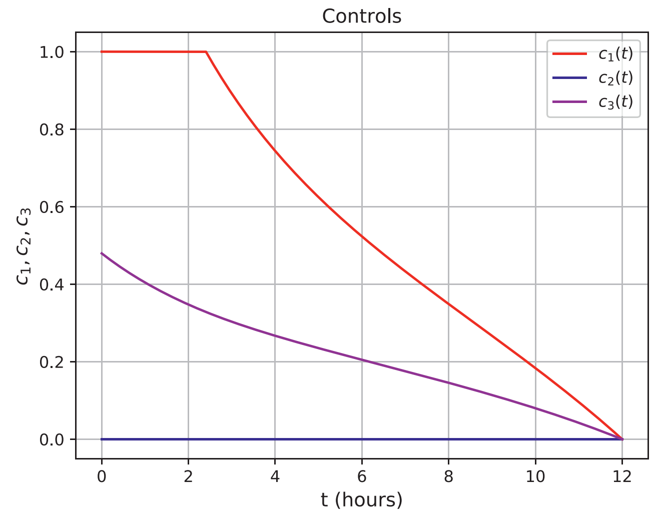
4.1.2. Strategy II: application of two controls

In this subsection we investigate the effect of employing two controls at a time. That is the application of either education and detention only, education and sacking only or detention and sacking only. Fig. 7 shows the control profiles for these activities.

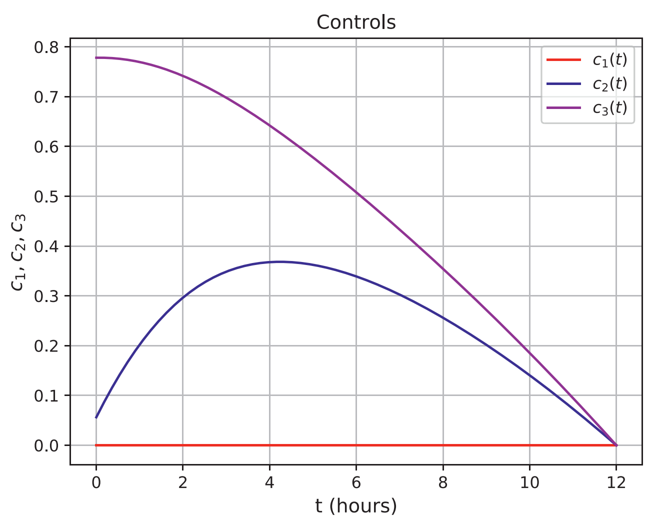
Fig. 8 a represents the effectiveness of applying both education and detention at a time. It is seen that these mixed strategies reduced the infected population by keeping the total population constant since both interventions do not take people out of the event premises. Also, Fig. 8b denotes the effectiveness of implementing both education and sacking. It is observed that the total number of infected population reduces when these control interventions are jointly applied. Since some infected population are being sacked, the total population decreases and infected population increases if there is no interventions. Finally, Fig. 8c indicates the effectiveness



(a) Control profile for $c_1 = c_2 \neq 0$ and $c_3 = 0$.



(b) Control profile for $c_1 = c_3 \neq 0$ and $c_2 = 0$.



(c) Control profile for $c_2 = c_3 \neq 0$ and $c_1 = 0$.

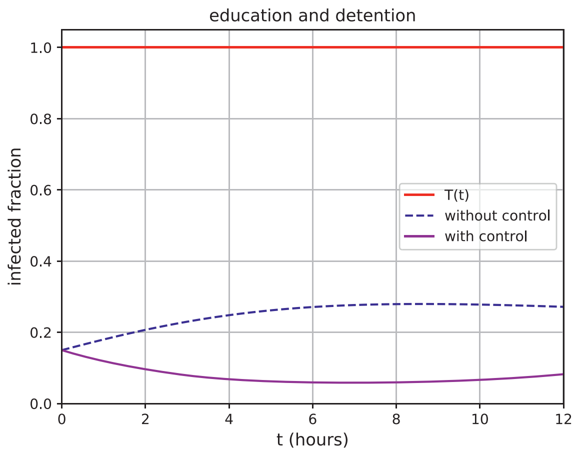
Fig. 7. Control profiles for implementing two controls.

of applying both detention and sacking as a control measure during an event. These strategies jointly reduce the total number of infected population and at the same time reduce the total population at the event premises.

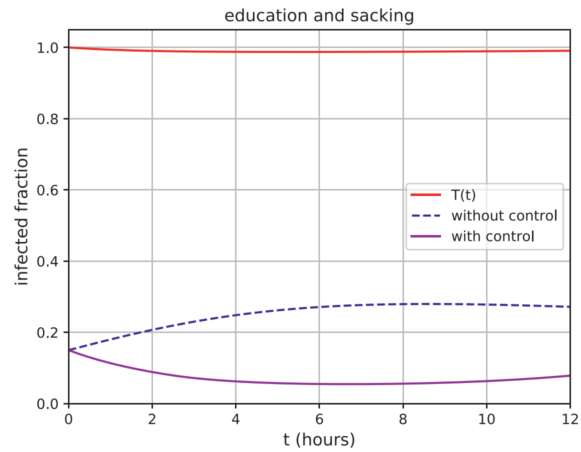
4.1.3. Strategy III: application of all controls

In this sub - section, we investigate the effect of applying all the three controls (education, detention and sacking) proposed in the paper. Fig. 9 shows the control profiles for the control functions $c_1 = c_2 = c_3 \neq 0$ with regards to the objective function (23). The numerical simulations here suggest that efforts should be placed on all the three control functions. However, more attention should be given to c_1, that is education. This is due to the fact that the control profile c_1 covers the greater portion of the upper bound of the time horizon and sharply drops to the final time indicating that c_1 is more effective and a sustainable intervention relative to c_2 and c_3 control functions.

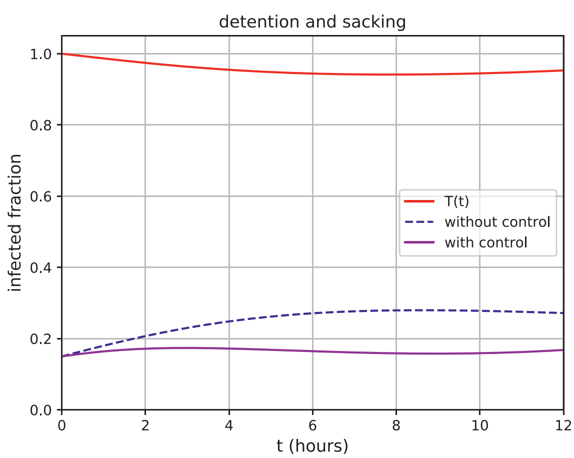
Fig. 10 represents the result obtained when we compare the model with and without controls. It is observed that, the controls have reduced the proportion of criminals causing more proportion of people to move into the susceptible compartment and some proportion moving to the “with intention” to commit crime compartment as shown in Fig. 10b. The population of infected (criminals) grow exponentially as shown in Fig. 10a but in Fig. 10b, it decline but never goes to zero. This conformed to the fact that it is difficult to totally eradicate crime from any event or festive period. Thus, it is necessary to sustain crime by keeping it at a minimal level and not to consider it as a situation that can be totally or fully avoided [8]. Also, the effectiveness of sacking as a control intervention is seen in Fig. 10b where some individuals were taken out of the event causing the total population to decrease at some particular time. Furthermore, in Fig. 10c, it is clearly observed that the infected population decreases and approaches zero when all the control interventions are implemented. It can further be seen that due to the interventions such as sacking and detaining, the total population reduces from the constant value of 1.



(a) Implementing both education and detention.



(b) Implementing both education and sacking.



(c) Implementing both detention and sacking.

Fig. 8. Effect of combining two control interventions on model (21).

Table 3
Computation of the controls basic reproduction number.

Controls	Control reproduction number
$c_1 = 1, c_2 = 0, c_3 = 0$	0.6672
$c_1 = 0, c_2 = 1, c_3 = 0$	1.0831
$c_1 = 0, c_2 = 0, c_3 = 1$	1.2063
$c_1 = 1, c_2 = 1, c_3 = 0$	0.0
$c_1 = 1, c_2 = 0, c_3 = 1$	0.4560
$c_1 = 0, c_2 = 1, c_3 = 1$	0.7465
$c_1 = 1, c_2 = 1, c_3 = 1$	0.0

We further employed the idea in Gervas et al. [25], Opoku and Afriyie [26] to analyse our result using the control reproduction number given by Eq. (22) and the parameter values stated in Table 2 to determine the control reproduction number for each control that has been administered. Comparatively, it can be observed from Table 3 that implementing only sacking or only detention is not effective compared to implementing only education as a control strategy. Because the control reproduction number for sacking only or detention only is given by (1.2062) and (1.08301) respectively, while the control reproduction number of implementing only education is given by 0.66719. From the basic concept of epidemiology, a reproduction number less than unity implies that crime can be eradicated from the event while when it is greater than unity, then it implies crime will persist in the event. We also observed that the application of two or all the three strategies provide the best results. This can be seen from the values obtained from the control basic reproduction number. Hence, it is advisable for event planners or organisers to implement two or all the control strategies under consideration during an event but if only one control strategy is to be applied then it has to do with educating participants on the awareness of criminal activities.

4.2. Cost effectiveness analysis

Based on the numerical results obtained in Section 4.1 we carry out the cost benefits attached to the implementation of the three control strategies. Following the idea in Bala and Zarkin [27,28], we make use of the incremental cost-effectiveness ratio (ICER) to determine the most effective cost strategy of the control strategies. Mathematically, ICER is implemented in the form

$$\text{ICER} = \frac{\text{Differences in costs of control strategies in } i \text{ and } j}{\text{Differences in infections averted by the control strategies in } i \text{ and } j}. \tag{27}$$

Recalling the simulation results of the optimality system and using the cost function $\frac{1}{2}W_1c_1^2$, $\frac{1}{2}W_2c_2^2$ and $\frac{1}{2}W_3c_3^2$ over time, we

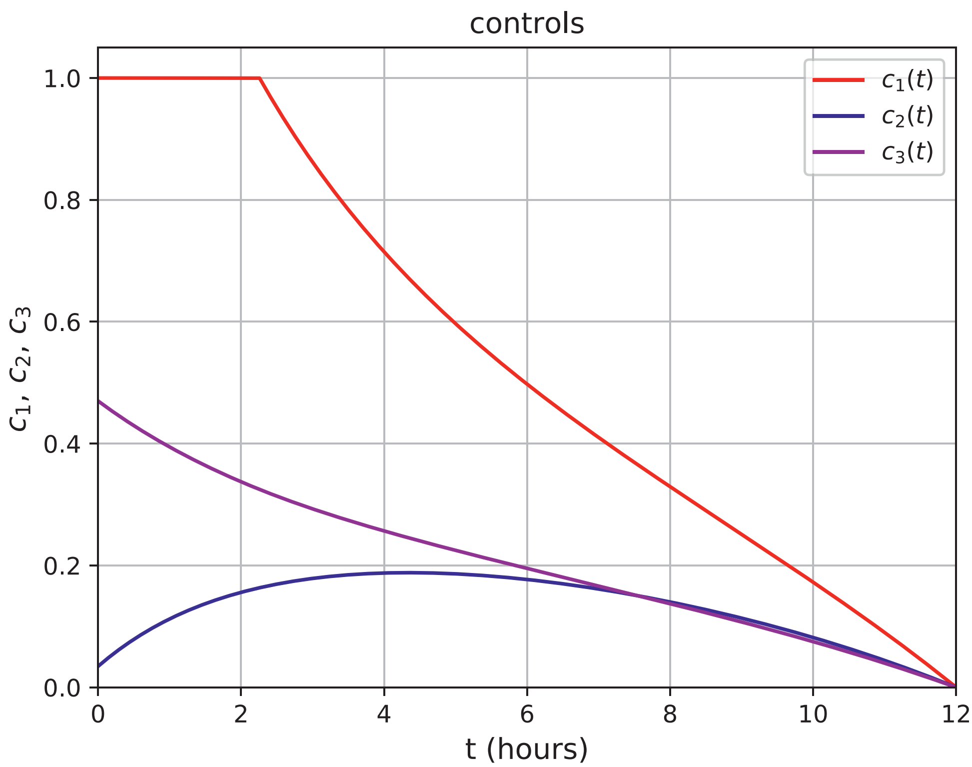


Fig. 9. Control profiles for $c_1 = c_2 = c_3 \neq 0$.

Table 4
Computation of total crime infection averted and its associated cost for each strategies.

Strategy	Description	TIA	Total cost (USD ($))	ICER
III	Education, Detention and Sacking	5016.5	0.57	0.000114
	(A) Education and Detention	4941.0	0.41	8.298×10^{-5}
II	(B) Education and Sacking	4774.01	0.41	8.588×10^{-5}
	(C) Detention and Sacking	4437.0	0.032	7.212×10^{-6}
	(a) Education	2647.9	0.025	9.441×10^{-6}
I	(b) Detention	1904.6	0.016	8.401×10^{-6}
	(c) Sacking	1295.1	0.016	9.441×10^{-6}

rank in descending order the intervention strategies showing it associated total cost (TC) and total infections averted (TIA) for pairwise comparison as indicated in Table 4.

From Table 4 we compare the various strategies in order to know the most cost effective intervention strategy. We begin by comparing the cost effectiveness of Strategy $I(a)$, $I(b)$ and $I(c)$.

$$ICER(I(a)) = 9.441 \times 10^{-6}.$$

$$ICER(I(b)) = \frac{0.016 - 0.025}{1904.6 - 2647.9} = 1.2109 \times 10^{-5}.$$

$$ICER(I(c)) = \frac{0.016 - 0.016}{1295.1 - 1904.6} = 0.$$

We observed that though implementing just a single strategy is mostly not advisable since it has little effect in eradicating crime, the cost attached to strategy $I(b)$ was found to be expensive followed by strategy $I(a)$. Strategy $I(c)$ was observed to have the least cost. The result shows that detaining an individual who have committed a crime or have the intention to commit a crime comes with the possibility of getting an additional security person to monitor the behaviour of this individual coming with an additional cost. However, sacking an individual with a criminal activity comes with a lesser cost.

In a similar manner, we compare the cost effectiveness of Strategy *II* (A, B and C) and *III*.

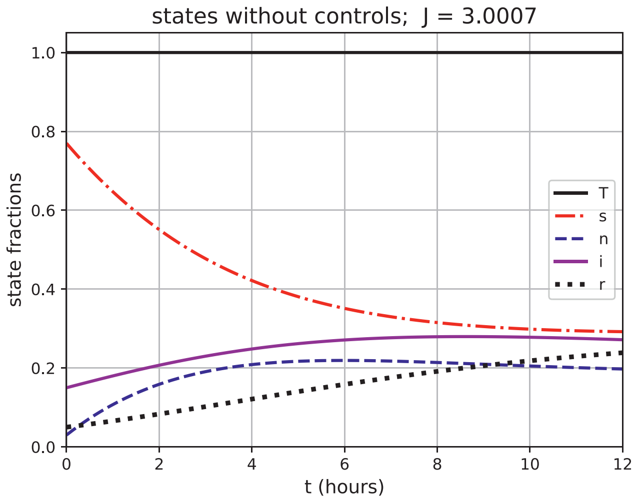
$$ICER(II(A)) = 8.298 \times 10^{-5}.$$

$$ICER(II(B)) = \frac{0.41 - 0.41}{4941 - 4774.01} = 0.$$

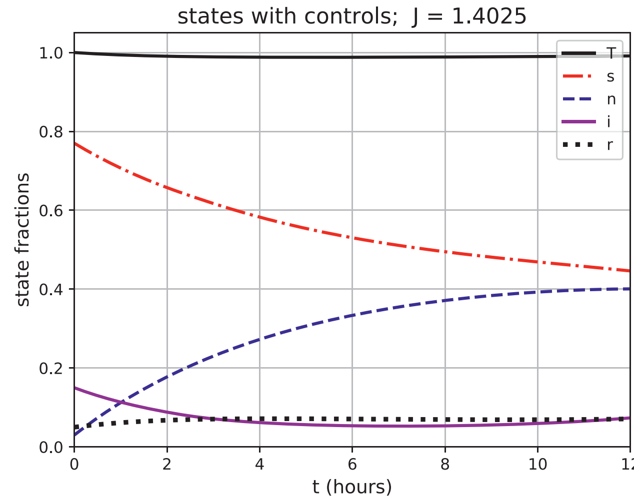
$$ICER(II(C)) = \frac{0.032 - 0.41}{4437 - 4474.01} = 1.122 \times 10^{-3}.$$

$$ICER(III) = \frac{0.41 - 0.57}{4941 - 5016.5} = 0.00212.$$

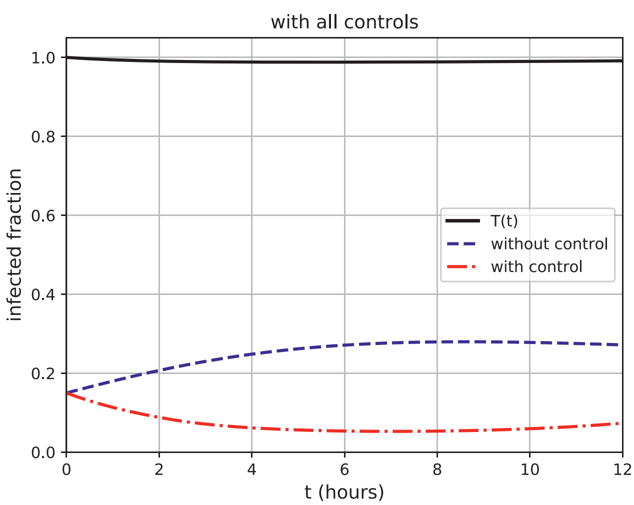
Comparing the cost of implementing two strategies, we observed that the most cost effective approach is strategy $II(B)$ while strategy $II(C)$ is expensive to implement. This has to do with the fact that in implementing strategy $II(C)$ additional personnel is required to successfully implement them. Moreover, we observe that implementing all three interventions (Strategy *III*) at once is the most expensive of all the applied intervention strategies.



(a) State variables without controls.



(b) State variables with controls.



(c) Implementing all intervention strategies on infected population.

Fig. 10. Effect of applying education, detention and sacking on model (21).

5. Conclusion

In this paper, a mathematical model consisting of four systems of nonlinear ordinary differential equations is formulated and used to study the dynamics of the spread of crime during an event. The threshold (basic reproduction number, R_0) was obtained which determines whether crime will persist during festive periods or will die off. The existence and stability of crime-free and crime endemic equilibrium was analysed. The stability equilibria switch at the bifurcation point when $R_0 = 1$. Our model (2) exhibits both backward and forward bifurcation depending on the length of the programme during an event. The requirement that $R_0 < 1$ is not enough and sufficient for crime elimination. An optimal control problem relative to model (2) was set up to minimise criminal activities during events. The control reproduction number was established to ascertain the best approach in minimising crime. Table 3 depicts the result. The significant discoveries of the behaviour of our model (21) were numerically checked. Examining the behaviour of our model without controls were compared with the model with control interventions to see the effect of these controls on the infected state variable. Furthermore, we investigated the cost effectiveness to determine the least and most expensive strategies by using ICER. From the pairwise result, we show that, the combination of education and sacking is the best cost-effective strategies in terms of cost.

Declaration of Competing Interest

Authors declare that they have no conflict of interest.

CRediT authorship contribution statement

Nicholas Kwasi-Do Ohene Opoku: Conceptualization, Supervision, Investigation, Validation, Formal analysis, Writing - original draft, Writing - review & editing. **Georg Bader:** Supervision, Investigation, Validation, Writing - review & editing. **Edem Fiatsonu:** Investigation, Writing - original draft, Validation.

Acknowledgements

The authors are thankful to the handling editor and the anonymous reviewers for their constructive comments and suggestions that greatly improved the presentation of the paper. The author's work did not receive any specific funding but was encouraged to perform the work to add to literature.

References

[1] Leonard G. Towards a legal history of american criminal theory: culture and doctrine from blackstone to the model penal code. Buffalo Crim Law Rev 2003;6(2):691–832.
[2] Lobonţ O-R, Nicolescu A-C, Moldovan N-C, Kuloğlu A. The effect of socioeconomic factors on crime rates in romania: a macro-level analysis. Econ Res-Ekon Istraž 2017;30(1):91–111.
[3] Machado H, Granja R. Forensic genetics in the governance of crime. Springer Nature; 2020. p. 114.
[4] Peskin M, Gao Y, Glenn AL, Rudo-Hutt A, Yang Y, Raine A. Biology and crime. In: Oxford handbook of criminological theory; 2012. p. 22–39.
[5] Walsh A, Beaver KM, editors. *The Ashgate research companion to biosocial theories of crime*. Ashgate Publishing, Ltd.; 2013.
[6] Johansson P, Kerr M. Psychopathy and intelligence: a second look. J Personal Disord 2005;19(4):357–69.
[7] Cohn EG, Rotton J. Even criminals take a holiday: instrumental and expressive crimes on major and minor holidays. J Crim Justice 2003;31(4):351–60.
[8] Chainey S, Ratcliffe J. GIS and crime mapping. John Wiley & Sons; 2013.
[9] González-Parra G, Chen-Charpentier B, Kojouharov HV. Mathematical modeling of crime as a social epidemic. J Interdiscip Math 2018;21(3):623–43.
[10] Berestycki H, Rodriguez N, Ryzhik L. Traveling wave solutions in a reaction-diffusion model for criminal activity. Multiscale Model Simul 2013;11(4):1097–126.
[11] Patel DM, Taylor RM. Social and economic costs of violence: Workshop summary. National Academies Press; 2012.
[12] Sooknanan J, Bhatt B, Comissiong D. Catching a gang—A mathematical model of the spread of gangs in a population treated as an infectious disease. Int J Pure Appl Math 2013;83(1):25–43.
[13] Saldana J, Aguareles M, Aviny A, Pellicer M, Ripoll J. An age-structured population approach for the mathematical modeling of urban burglaries. SIAM J Appl Dyn Syst 2018;17(4):2733–60.
[14] Raimundo SM, Yang HM, Massad E. Contagious criminal career models showing backward bifurcations: implications for crime control policies. J Appl Math 2018;2018.
[15] Backman O, Edling C. Review essay: mathematics matters: on the absence of mathematical models in quantitative sociology. Acta Sociol 1999;42(1):69–78.
[16] Jones JH. Notes on R0. Califonia: Department of Anthropological Sciences; 2007.
[17] Van den Driessche P, Watmough J. Further notes on the basic reproduction number. In: Mathematical epidemiology. Berlin, Heidelberg: Springer; 2008. p. 159–78.
[18] Brauer F. Backward bifurcations in simple vaccination models. J Math Anal Appl 2004;298(2):418–31.
[19] Castillo-Chavez C, Song B. Dynamical models of tuberculosis and their applications. Math Biosci Eng 2004;1(2):361.
[20] Opoku N, Nyabadza F, Gwasira E. Modelling cervical cancer due to human papillomavirus infection in the presence of vaccination. Open J Math Sci 2019;3(1):216–33.
[21] Pontryagin LS, Boltyanskii V, Gamkrelidze R, Mishchenko E. The mathematical theory of optimal processes. New York: Translated by KN Trirogoff; 1962.
[22] Lenhart S, Workman JT. Optimal control applied to biological models. CRC press; 2007.
[23] Bader G, Ascher U. A new basis implementation for a mixed order boundary value ode solver. SIAM J Sci Stat Comput 1987;8(4):483–500.
[24] Ugwuishiwu CH, Sarki D, Mbah G. Nonlinear analysis of the dynamics of criminality and victimisation: a mathematical model with case generation and forwarding. J Appl Math 2019;2019.
[25] Gervas HE, Opoku NK-DO, Ibrahim S. Mathematical modelling of human African trypanosomiasis using control measures. Comput Math Methods Med 2018;2018.
[26] Opoku NK-DO, Afriyie C. The role of control measures and the environment in the transmission dynamics of cholera, 2020. Hindawi; 2020.
[27] Bala MV, Zarkin GA. Application of cost-effectiveness analysis to multiple products: a practical guide. Am J Manag Care 2002;8(3):211–20.
[28] Levin HM, McEwan PJ. Cost-effectiveness analysis: Methods and applications, Vol. 4. Sage; 2000.