

Courses For 2025/2026 Academic Year

Course Code	Lecturer	Course Title
Skills Phase		
Block S1 (25th August - 12th September 2025)		
MSCI 505/605	Bernard Bainson	Mathematical Problem Solving
MSCI 503/603	Eyram Schwinger	Scientific Computing with Python
Engagement Week (15th Sept - 19th Sept 2025)		
Block S2 (22nd September - 10th October 2025)		
MSCI 507/607	Eugene Adjei & Rhoda Hawkins	Physical Problem Solving & Modelling
MSCI 531/631	Patrick Dorey & Nick Monk	Ordinary Differential Equations
Block S3 (13th - 31st October 2025)		
MSCI 509/609	Prince Osei	Introduction to Linear Algebra
MSCI 561/661	Richard Minkah	Probability & Statistics with R
Reading week (3rd Nov - 7th Nov 2025)		
Review Phase (Elective courses)		
Block R1 (10th - 28th November 2025)		
MSCI 560/660	Rosemary Harris	Stochastic non-equilibrium processes
MSCI 574/674	Ulrich Krähmer & Angela Tabiri	Galois theory
MSCI 562/662	Issa Karambal	Machine Learning
Block R2 (1st - 19th December 2025)		
MSCI 564/664	Djoko Wirosoetisno	Dynamical Systems
MSCI 565/665	Abebe Geletu	Optimization: Theory, Methods and Applications
MSCI 553/653	Franck Kalala Mutombo	Network Science for Big Data

Christmas break (22nd Dec 2025 - 4th Jan 2026)

Block R3 (5th - 23rd January 2026)		
MSCI 558/658	Olivier Pamen	Stochastic Analysis
MSCI 535/635	Hallowed Olaoluwa	Functional Analysis
MSCI 572/672	Eric Chitambar	Quantum Information Processing

Block R4 (26th January - 13th February 2026)		
MSCI 594/694	Antonio Duarte Pereira Junior	Quantum Mechanics
MSCI 520/620	Peter Webb	Group Theory
MSCI 571/671	Zuzana Masárová	Topics in computational Geometry
MSCI 524/624	Josef Tkadlec	Evolutionary graph theory

Reading week (16th - 20th Feb 2026)

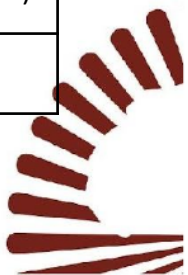
Block R5 (23rd February - 13th March 2026)		
MSCI 596/696	Nick Monk	Mathematical Biology
MSCI 522/622	Dan Kucеровsky	Cryptography
MSCI 556/656	Phil Knight & Heather Yorston	Numerical Analysis with Python

Block R6 (16th March - 3rd April 2026)		
MSCI 526/626	Tannie Liverpool	Statistical Physics
MSCI 532/632	John Parker	Hyperbolic Geometry
MSCI 587/687	Hans Nordstrom	Category theory & applications in AI

Research Phase (6th April - 19th June 2026)

MSCI 500/600		Research Project (Report deadline 5th June, Orals 15th-19th June)
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Graduation (27th June 2026)



Course Descriptions

SKILLS COURSES

MSCI 505/605: Mathematical Problem Solving

a. Objective: The objective is to: illustrate, gain practical experience and study different approaches to problem solving and research.

b. Content: This course considers a variety of elementary, but challenging problems in different branches of pure mathematics. Investigations, comparisons of different methods of attack, literature searches, solutions and generalisations of the problems will arise in discussions in class.

c. Reading materials include:

1. Martin J. Erickson and Joe Flowers, (1999). *Principles of Mathematical Problem Solving*, Prentice Hall.
2. L.C. Larson, (1983). *Problem Solving through Problems*. Problem Books in Mathematics, Springer-Verlag.
3. Mary Boas, *Mathematical Methods in the Physical Sciences*, Wiley (2nd Edition 1983)
4. W. A. Wickelgren (1995) *How to Solve Mathematical Problems*, Dover Books

MSCI 503/603: Scientific Computing with Python

a. Objective: Upon successfully completing this course, students will be able to: “do something useful with Python”.

1. Identify/characterise/ define a problem
2. Design a program to solve the problem
3. Create executable code
4. Read most Python code
5. Write basic unit tests

b. Content: We will cover (1) programming (2) for research (3) with Python, IN THAT ORDER. First, we will develop the practical skill of programming: expressing a task (1) in a formal language, (2) in a way that is useful, and doing so (3) quickly and (4) flexibly.

Second, we will focus on tasks that are most pertinent to research (rather than more general purpose programming, like making a text editor or music player), e.g.:

- reading / writing data
- transformation / subsetting of data
- bulk analysis of data
- enumeration of states
- approximation and simulation of models
- visualisation of results

Finally, we will cover how to realise these tasks in a particular language - Python - using tools associated with that language (e.g., the command line interpreter, scripts, Python notebooks).

c. Reading materials include:



1. Programming in Python 3: A Complete Introduction to the Python Language
(<https://www.safaribooksonline.com/library/view/programming-in-python/9780321699909/>)
2. Data Science from Scratch: First Principles with Python
(<https://www.safaribooksonline.com/library/view/data-science-from/9781491901410/>)
3. Python Data Analytics: Data Analysis and Science Using Pandas, matplotlib, and the Python Programming Language
(<https://www.safaribooksonline.com/library/view/python-data-analytics/978148420958/>)
4. Problem Solving with Python (<https://problemsolvingwithpython.com/>)
5. George Grätzer, (2007), *More Math into LaTeX*, 4th edition, Springer, ISBN 978-0387322896.
6. Stefan Kottwitz, (2011), *LaTeX Beginner's Guide*. Packt Publishing.
7. LaTeX Wikibook: <https://upload.wikimedia.org/wikipedia/commons/2/2d/LaTeX.pdf>

MSCI 507/607: Physical Problem Solving & Modelling

a. Objective:

The students will be able to:

1. describe and explain physics concepts including knowing where and when they apply;
2. apply physics concepts when solving problems and examining physical phenomena
3. apply concepts in new contexts and combine concepts, when analysing a situation.
4. develop a good functional understanding of physics

b. Content: The course has three components:

1. Overview lectures outlining main branches and key concepts in physics such as space and time, force, energy, fields, temperature, entropy, quantum theory.
2. Problems solving in small groups, focusing on problems which can be addressed with minimal technical prerequisites. Use of scaling and dimensional analysis, order of magnitude estimates, simple programming if possible.

c. Reading materials include:

1. Moursund's book, *Brief Introduction to Roles of Computers in Problem Solving*,
2. Kenneth Heller, Patricia Heller, (2010) University of Minnesota. *Cooperative Problem Solving in Physics A User's Manual*.
3. Mary Boas, *Mathematical Methods in the Physical Sciences*, Wiley (2nd Edition 1983)
4. Alastair Rae, *Quantum Mechanics*
5. Michael Conterio, *How To Solve Physics Problems* (2019)

MSCI 558/658: Ordinary Differential Equations

a. Objectives: Students should be able to:

State, prove and apply the fundamental existence and uniqueness results for ODEs and qualitative techniques for the study of behaviour of solutions for ODEs.

b. Content: This course is an introduction to the theory and applications of differential equations, covering the following topics

1. Classification of ODEs and elementary methods of integration
2. Existence and uniqueness results for systems of ODEs



3. Applications of second order systems
4. Phase diagrams, equilibrium points and the concept of stability

c. Reading materials include:

Arnold V. I., (1973), *Ordinary differential equations.*, MIT Press.

Barreira L. and Valls C., (2010), *Ordinary differential equations: qualitative theory.*AMS,ISBN: 978-0-8218-8749-3.

Hirsch M. W. and Smale S., (2004), *Differential equations, dynamical systems and an introduction to chaos*", Academic Press/Elsevier, ISBN: 978-0-12-349703-1.

Bernd J. Schroers, (2011). *Ordinary differential equations: a practical guide*, Cambridge, ISBN: 978-1-107-69749-2, AIMS Library Series.

MSCI 509/609: Introduction to Linear Algebra

a. Objectives:The course teaches students concepts in linear algebra in order for students to:

- Undertake research using the tools from the course
- Strengthen the theoretical foundations of students to bridge the gap between pure and applied mathematics
- Apply linear algebra to modern topics in data science, artificial intelligence and real world problems in general

b. Content:

- Fields and Vector spaces over fields
- Basis of a vector space
- Linear maps
- Matrices
- Solving systems of linear equations
- Correspondence between linear maps and matrices
- Rank-Nullity Theorem
- Eigenvalues and eigenvectors
- Invariant subspaces and upper triangular matrices
- Generalised eigenvectors
- Jordan form and Jordan block
- Inner Product Spaces
- Spectral Theorem and Quadratic Forms
- Multilinear Algebra
- Special Topics and Research Applications

c. Reading materials include:

1. Linear Algebra Done Right by Sheldon Axler (4th Edition, Springer, 2024)
2. A Second Course in Linear Algebra by Stephan H. Ramon (Cambridge University Press, 2017)
3. Introduction to Linear Algebra by Gilbert Strang, (6th Edition 2023)



MSCI 561/661: Probability and Statistics with R

a. Objective: The course objective is to familiarise students with a mathematically rigorous approach to probability and statistics and to handle problems both analytically and computationally with an introduction to using R.

By the completion of the course, students will be able to:

1. Write efficient and transparent programs in R
2. Produce clear and effective graphical descriptions of data
3. Import, export and manipulate datasets
4. Analyze data using descriptive and inferential statistics
5. Analyze data through model fitting
6. Use R packages to efficiently manage and model univariate and multivariate data

b. Content: This course is designed to inspire students to consider the fundamental principles of statistical science as important to wider fields of life and to understand these key principles as building blocks for further statistical courses. Some students may enroll in AIMS with limited experience of statistics, or of data science, but through lectures and practicals they can all gain confidence in analysing data, and presentation of results and report writing. We will show how this skill is used in real life to provide evidence for key decision-making which can change policy and contribute to improving health, education and other standards of living in a Country. The main aim of the course is to introduce the students to probability and the theory at the heart of statistics, teaching them methods of data analysis giving students practical applications in solving real-world problems. The course includes practical workshops to consider: the design, collection

and management of data, as well as the analysis, interpretation and presentation of results. The overall objective of the course is to equip the learners with the skills and knowledge of using data and statistical methods to solve real statistical problems. The Course includes Probability and probability generating functions, conditional probability and Bayes Theorem; a gentle introduction to R for descriptive statistics and graphics leads to definitions of hypothesis testing and confidence intervals (as the “wings of uncertainty”) correlation coefficient, least squares regression and a brief introduction to non-parametric methods and Bayesian Statistics. The course leads to a group project with analysis of real data chosen by students -to answer a question of their interest and choice on African Countries. Throughout the course, the students experience steps in the process of solving problems. The skills taught in this course are useful to a wide range of disciplines, with a high demand in the real-world.

This course provides training in R/Rstudio and the statistical concepts most commonly used in practice. This course will introduce theoretical and practical concepts from the basics of statistical computing to statistical modeling. The course focuses on developing programming skills with R/Rstudio and familiarizing students with a mathematically rigorous approach to probability and statistics. Key topics include introduction to R, mathematical computations, graphics, descriptive statistics, probability distributions, hypothesis testing, linear regression, numerical methods, and bootstrapping. The course also features hands-on training with both simulated and real data sets.

c. Reading materials include:

1. JA Crawshaw *A Concise Course in A-level Statistics with worked examples*
2. SE Hodge and ML Seed *Statistics and Probability*
3. G Grimmett and D Welsh *Probability: an Introduction*.



4. W Feller: *An Introduction to Probability Theory and its Applications*
5. D Williams *Weighing the Odds: a Course in Probability and Statistics*.
6. De Veaux, R, Velleman, P., and Bock, D. (2014). *Introductory Statistics with DVD*. 4th ed. New York: Pearson/Addison-Wesley.
7. Craine III, William B., DeVeaux, R., Velleman, P. and Bock, D. (2014). *Introductory Statistics. Student Solutions Manual*. 4th ed. New York: Pearson/Addison-Wesley.
8. Jeffrey O. Bennett, William L. Briggs, and Mario F. Triola, (2009). *Statistical Reasoning For Everyday Life*: 3rd ed. Pearson
9. STAT 220/230/240 COURSE NOTES by Chris Springer, Revised by Jerry Lawless, Don McLeish and Cynthia Struthers.
10. STATISTICS 231 COURSE NOTES, 2016 Edition, Department of Statistics and Actuarial Science, University of Waterloo,
11. Rizzo, M. L. (2007). *Statistical computing with R*. CRC Press.
12. Kabacoff, R. I. (2010). *R in Action*. Manning.
13. Efron, B., & Tibshirani, R. J. (1994). *An introduction to the bootstrap*. CRC press.
14. Gareth, J., Witten, D., Hastie, T., and Tibshirani, R. (2017). *An Introduction to Statistical Learning with Applications in R*. Springer.
15. Zaki, M. J. and Meira, Jr, W. (2014). *Data Mining and Analysis: Fundamental Concept and Algorithms*. Cambridge University Press.
16. *Introduction to Probability, Statistics, and Random Processes* by Hossein Pishro Nik
17. *Statistics, an introduction using R* by Michael J. Crawley
18. *Modern Statistics with R*, Måns Thulin
19. *Learning Statistics with R* by Danielle Navarro
20. *Introductory Statistics with R* by Peter Dalgaard
21. *Causal Inference in Statistics: A Primer* (Pearl, Glymour, Jewell, 2016).
22. *Elements of Causal Inference: Foundations and Learning Algorithms* (by Peters, Janzing and Schölkopf)
23. Hernán MA, Robins JM (2020). *Causal Inference: What If*. Boca Raton: Chapman & Hall/CRC
24. *More advanced: Causality* (Pearl, 2009)

REVIEW COURSES

MSCI 560/660 Stochastic non-equilibrium processes

a. Objective: Upon successfully completing the course, students will have gained a flavour of some mathematical methods used in the study of stochastic nonequilibrium processes and be able to apply them to simple models.

b. Content: Stochastic processes, in which the values of variables change probabilistically in time, are of considerable importance in mathematics with widespread applications in physics, chemistry, biology, and finance. For *nonequilibrium* dynamics, time-reversal symmetry is broken and there are typically probability currents corresponding, for example, to the flow of particles/material in a physical setting: the fluctuations and symmetries of those currents are a topical area of statistical mechanics research. This course will introduce some of the mathematical methods (such as large deviation theory) used in studying stochastic nonequilibrium processes and illustrate them with paradigmatic statistical physics models. We will start gently with single-particle Markov processes (i.e., biased random walks!) and build-up towards many-particle and, time permitting, non-Markov processes. The maths will

mainly be presented at a nonrigorous level, with the focus on practical calculations supplemented by computational experiments.

c. Reading materials include:

1. An Introduction to Stochastic Processes and Nonequilibrium Statistical Physics, Horacio S Wlo, Roberto R Deza, Juan M Lopez, 2012
2. Stochastic Processes in Nonequilibrium Systems, L. Garrido, P. Seglar, P. J. Sheperd, Springer, 1978
3. Nonequilibrium Statistical Mechanics Basic concepts, models and applications, Alessandro Sarracino, Andrea Puglisi and Angelo Vulpiani, IOP Publishing, 2025

MSCI 574/674: Galois Theory

a. Objectives: By the end of the course students should be familiar with the basics of abstract algebra, groups and Galois theory.

b. Content:

I will begin with classical Galois theory including constructions by ruler and compass up to the (non)solvability of quintic equations. If required, all the necessary abstract algebra (groups, rings, fields, linear algebra) will be recalled. If time permits, I'll add lectures on noncommutative division algebras and Hopf-Galois extensions at the end to connect the classical theory to current research.

c. Reading materials include:

1. Serre's Linear Representations of Finite Groups
2. Alperin and Bell's Groups and Representations.
3. I.M. Paramanov, Symmetries in Mathematics.
4. Deng, B. and Du, J. et al(2008), Finite Dimensional Algebras and Quantum Groups, Rhode Island, American Mathematical Society.
5. Grillet, P.A.(2007), Abstract Algebra, New York, Springer-Verlag N.Y.Inc.
6. Hatcher, A.(2002), Algebraic Topology, Cambridge, Cambridge University Press.

MSCI 562/662: Machine Learning

a. Objectives: Students should be able to:

1. describe the basic concepts of machine learning;
2. describe some machine learning algorithms including, classification, regression, clustering
3. recognize complex patterns and make intelligent decisions based on data;
4. determine when these methods are required;
5. study the techniques and use in their research.

b. Content: Introduction to Pattern recognition and application to bioinformatics, speech recognition, finance, medical diagnosis and artificial vision and their classification. MRI interpretation, regression and data dimensionality reduction. Related computer projects are investigated using SciPy and NumPy or their extensions.

c. Reading materials include:

1. R. Duda, P.Hart (2000). *Pattern Classification 2nd Ed.* Wiley-Interscience
2. C. Bishop (2006). *Pattern recognition and machine learning*, Springer.
3. Andriy Burkov (2019) *The Hundred-Page Machine Learning Book*
4. Goodfellow et al, (2016) *Deep Learning*, MIT Press



MSCI 564/664: Dynamical Systems

a. Objectives: By the end of the course students will be able to describe the properties of solutions to differential equations and perform relevant calculations.

b. Content:

This course studies properties of the solutions of differential (and some difference) equations, mostly in one and two space dimensions, with emphasis on their qualitative behaviour. We plan to cover the following topics:

- Review of the existence and uniqueness (Picard-Lindelof) results.
- Linearisation (Hartman-Grobman) theorem; stability and Lyapunov functions.
- Bifurcation theory: dependence of solutions on parameters.
- Orbits and periodic solutions.
- Planar (2d) systems: Poincare-Bendixson and index theorems.
- Lorenz system and introduction to chaotic dynamics (time permitting).

c. Reading materials include:

1. Arnold V. I., (1973), *Ordinary differential equations.*, MIT Press.
2. Hirsch M. W. and Smale S., (2004), *Differential equations, dynamical systems and an introduction to chaos*, Academic Press/Elsevier, ISBN: 978-0-12-349703-1.
3. Bernd J. Schroers, (2011). *Ordinary differential equations: a practical guide*, Cambridge, ISBN: 978-1-107-69749-2, AIMS Library Series.

MSCI 565/665: Optimization: Theory, Methods and Applications

a. Objectives: By the end of the course students will be able to describe and use standard mathematical optimization algorithms.

b. Content:

This course is designed to teach modern and standard mathematical optimization algorithms for the solution of complex real-life problems in the presence of process data. The course-coverage focuses on gradient-based optimization algorithms that are efficient for the solution of problems related to pattern recognition, feature extraction, prediction, classification, object detection, object reconstruction, compressed sensing, neural-network-based modeling and applications. The following topics will be covered:

- Basics of mathematical optimization
- Optimization algorithms for unconstrained optimization problems
- Constrained optimization methods for data-driven applications

c. Reading materials include:

1. Steve Boyd, Sven Vandenberg: Convex optimization. Cambridge University Press, 2004.
2. Jorg Nocedal, Stephen J. Wright: Numerical Optimization, 2nd Ed. Springer Verlag, 2006.
3. Aurelien Geron: Hands-on Machine Learning with Scikit-learn, Keras and TesnorFlow, 2nd Ed., OReilly, 2019.
4. Aston Zhang, Zachary C. Lipton, Mu Li, and Alexander J. Smola: Dive into Deep Learning. Cambridge University Press; 1st edition, 2023.
5. Ian Goodfellow, Yoshua Bengio, and Aaron Courville: Deep Learning. MIT Press, 2016.



MSCI 553/653: Network Science for Big Data

a. Objectives: This course explores the foundations and applications of network science in the context of big data. Students will learn how to model, analyze, and interpret complex networks that arise in various fields, such as social networks, biological systems, transportation, and communication. The course will cover key concepts, including graph theory, network topology, centrality measures, community detection, and network dynamics. Through a combination of theoretical lectures, practical programming exercises, and real-world case studies, students will develop the skills needed to extract meaningful insights from large-scale network data. Emphasis will be placed on scalable methods and tools for handling and analyzing big data, enabling students to address challenges in modern data-intensive applications.

By the end of the course, students/participants will be equipped with the knowledge and techniques to apply network science to solve problems in diverse domains, contributing to research and industry innovation. For this course we will be using Python (NetworkX and Igraph (optional)). For visualization will use packages like Gephi, Pajek, Cytoscape

Learning Outcomes:

- Understand the structure and dynamics of complex networks.
- Apply computational techniques to analyze networks at scale.
- Derive actionable insights from network data for various real-world scenarios.

b. Content:

1. Foundations of Network Science (introduction to graph theory and networks)
 - Formal definitions
 - Basics of graph theory: nodes, edges, adjacency matrices.
 - Creating network in Python
 - Key network properties: centrality measures, clustering coefficients, degree distribution and connectivity
 - Assortativity and mixing patterns in networks
 - Network models
 - Advanced use cases of network science
2. Big Data Integration
 - Handling large-scale network data using distributed systems like Apache Spark and Hadoop.
 - Data preprocessing and transformation techniques for network datasets.
 - Wrangling Data into Networks
3. Algorithms and Techniques
 - Network clustering and community detection.
 - Algorithms for community detection (e.g., Louvain, Girvan-Newman)
 - Dynamic networks: temporal analysis and evolving network structures.
 - Pathfinding and shortest paths in large graphs.
 - Machine Learning for Networks
4. Applications
 - Social network analysis: detecting influential nodes and communities.
 - Biological networks: understanding protein interaction networks.
 - Urban networks: optimizing transportation and communication systems.
 - Demographic Data, Ecological Data
5. Case Studies and Projects
 - Application to domains like epidemiology, e-commerce, and finance.
 - Capstone project to analyze and visualize a large-scale network dataset
6. Next-generation Network Tools for Complex, Big Data (optional*)

c. Reading materials include:**Books**

1. "Networks, Crowds, and Markets: Reasoning About a Highly Connected World"
Authors: David Easley, Jon Kleinberg. Overview: A comprehensive introduction to network science, including economic and social applications.
2. "Networks: An Introduction" Author: Mark Newman. Overview: A foundational textbook covering graph theory, network modeling, and structural analysis.
3. "Dynamic Networks and Cyber-Security" Authors: Niall Adams, Nick Heard. Overview: Explores dynamic networks with applications to big data and cyber-security.
4. "Big Data and Social Science: A Practical Guide to Methods and Tools" Authors: Ian Foster et al. Overview: Focuses on tools and techniques for working with large-scale network data in social science contexts.
5. "Graph Representation Learning" Authors: William L. Hamilton. Overview: Covers advanced topics like graph embeddings and deep learning on networks
6. "Modern Graph Theory Algorithm with Python: Harness the power of graph algorithms and real-world network applications using Python." Authors: Colleen M. Farrelly , Franck Kalala Mutombo

Articles

1. Barabási, A. L. (2009). "Scale-Free Networks: A Decade and Beyond." *Science*, 325(5939), 412-413. Discusses the ubiquity of scale-free networks in real-world systems.
2. Newman, M. E. J. (2003). "The Structure and Function of Complex Networks." *SIAM Review*, 45(2), 167-256. A foundational review article on network topology and applications.
3. Leskovec, J., Rajaraman, A., & Ullman, J. D. (2014). "Mining of Massive Datasets." Focuses on algorithms and methods for large-scale graph and network analysis.
4. Fortunato, S. (2010). "Community Detection in Graphs." *Physics Reports*, 486(3-5), 75-174. A detailed survey of community detection methods.
5. Vespignani, A. (2012). "Modelling Dynamical Processes in Complex Socio-Technical Systems." *Nature Physics*, 8(1), 32-39. Explores network dynamics and their relevance to big data.

Journals

1. *Journal of Complex Networks*. Topics: Theoretical and applied aspects of complex network science.
2. *Nature Communications*. Topics: High-impact research in network science and interdisciplinary applications.
3. *Physical Review E*. Topics: Statistical physics of networks, including structural and dynamic properties.
4. *Social Networks*. Topics: Empirical studies and computational models in social network analysis.
5. *IEEE Transactions on Network Science and Engineering*. Topics: Advanced algorithms and applications for large-scale networks.
6. *Journal of Big Data*. Topics: Methods and case studies for handling large-scale network data.

MSCI 558/658: Stochastic Analysis

a. Objectives: By the end of the course students will have a basic foundation on stochastic analysis

b. Content: The aims at giving basic foundation on stochastic analysis. It will introduce the students to stochastic analysis with emphasis on Brownian motion, Ito's integral, Ito's formula



and applications and stochastic differential equations. The theoretical results will be illustrated with numerical examples. If time allows, applications will be made to mathematical finance.

Mathematical Preliminaries: Measure theory, probability space, random variable, expectation, filtration, conditional expectation, martingales, stopping time.

Stochastic integral: Brownian motion and properties, construction of Ito's integral, properties of Ito's integral, extension of Ito's integral, Ito's formula and applications, Girsanov Theorem, Martingale representation Theorem.

Stochastic differential equations: Global existence and uniqueness; diffusions and the PDE connection; Feynman-Kac representation.

Application to Mathematical Finance: risk-neutral pricing; derivative securities.

c. Reading materials include:

1. P. Baldi. Stochastic Calculus: An Introduction through Theory and Exercises. Springer Universitext, 2017.
2. I. Karatzas and S. E. Shreve. Brownian Motion and Stochastic Calculus. Springer-Verlag, New York, 1988.
3. B. Øksendal. Stochastic Differential Equations: An Introduction with Applications. Springer, Berlin, Heidelberg, New York, 2003.
4. Diffusions, Markov Processes and Martingales, L. C. G. Rogers, David Williams Cambridge University Press (2000) ISBN:9780511805141

MSCI 535/635: Functional Analysis

a. Objective: At the end of this course, students should be able to:

1. describe properties of normed linear spaces and construct examples of such spaces;
2. extend basic notions from calculus to metric spaces and normed linear spaces;
3. apply the Cauchy-Schwartz inequality to the derivation of other inequalities;
4. prove that a given space is a Hilbert or Banach space and
5. state and prove the Hahn-Banach theorem

b. Content: Normed linear space, Banach space, Bounded linear maps, Inner product space, Hilbert Space, Projection theorem, Reiss representation theorem; Measurable sets, Lebesgue Integral, Lpspaces; Sobolev spaces.

Mode of delivery: Lectures and presentation by students and practical computer work.

c. Reading materials include:

1. E. Kreyszig, (1978). *Introductory Functional Analysis with Applications*, Wiley.
2. Walter Rudin, *Functional Analysis*, McGraw Hill, ISBN: 0070542368.
3. Lawrence W. Baggett, Marcel Dekker. *Functional Analysis: a primer*, ISBN: 0824785983.
4. Peter Lax. *Functional Analysis*, Wiley, ISBN: 0471556041.

MSCI 572/672: Quantum Information Processing

a. Objectives: The Quantum Information Process course is treating the theory of a collection of the central topics necessary to understand why and how quantum computers function and what problems they are able to solve.

b. Content: The topics include quantum computational complexity and the quantum circuit model, powerful quantum algorithms such as Shor's algorithm for integer factorization or Grover's algorithm for unstructured search. Studying these topics will give insights into why quantum computers can outperform any type of 'classical' computers in certain, very specific tasks. However, our attention will also be drawn to difficulties arising when trying to build a quantum computer – and how to overcome them using quantum error correction.



The course is aimed at the basics of quantum information processing, so that after this course a participant is equipped with the skills to embark on further investigations of advanced topics. Structure of the course: Preliminaries (mathematical basics), Computational Complexity, Grover's Algorithm, The Quantum Fourier Transform, Period Finding and Integer Factorization, Quantum Error Correction, The Stabilizer Formalism, Fault-Tolerant Quantum Information Processing, if time permits: Hamiltonian Simulation.

c. Reading materials include:

1. Quantum Computation and Quantum Information, Michael Nielsen and Isaac Chuang, Cambridge University Press, 2000
2. An Introduction to Quantum Computing, Phillip Kaye, Raymond Laflamme, Michele Mosca, Oxford University Press, 2007
3. Quantum Information Theory, Mark Wilde, Cambridge University Press, 2013.
4. Quantum Information Processing Theory and Implementation, Janos A. Bergou, Mark Hillery, Mark Saffman, Springer 2021.

MSCI 594/694: Quantum Mechanics

a. Objectives: By the end of this course students should be able to:

1. describe the important principles of quantum theory
2. discuss quantum phenomena
3. be comfortable with the major mathematical representations

b. Content: In this course I will teach the basics of Quantum Theory. The main focus will be the introduction of the essential concepts in Quantum Theory and to substantiate them with their mathematical formulation. The topics I will cover are as follows:

1. Review of classical mechanics [Lecture 1]
2. Mathematical formalism of quantum mechanics [Lectures 1, 2]
3. Postulates of quantum mechanics [Lecture 3]
4. One dimensional problems [Lecture 4]
5. Harmonic oscillator [Lecture 5]
6. Angular momentum [Lecture 6]
7. Spin [Lecture 7]
8. Time independent perturbation theory [Lecture 8]
9. Heisenberg picture, path integrals [Lecture 9]

In particular, I will discuss the physics and mathematical formalism of two-level systems, thoroughly, to establish a sound basis. In order to consolidate the understanding of these ideas, I introduce basic examples for physical phenomena that clarify these concepts. Additionally, I will let the students calculate instructive examples in class themselves. for more advanced students.

c. Reading materials include:

1. Alastair Rae Quantum Mechanics ISBN: 1584889705, 9781584889700
2. Griffiths, David J. (1995). Introduction to quantum mechanics. ISBN 978-0-13-124405-4. OCLC 984428006
3. Aaronson, S. (2013) Quantum Computing since Democritus, Cambridge University Press.
4. Rieffel, E. and Polak, W. (2011) Quantum Computing- a gentle introduction, Massachusetts Institute of Technology.
5. Dimock, J. (2011) Quantum Mechanics and Quantum Field Theory: A Mathematical Primer 283 p., Cambridge University Press.
6. Mermin, N. David (2007). *Quantum Computer Science: An Introduction*. Cambridge University Press.
7. E. Buks, Quantum Mechanics Lecture Notes, Technion (2012).
8. P. Goldbart, Course Notes, University of Illinois (1996).



9. R. Shankar, Principles of Quantum Mechanics, Plenum (1994).

MSCI 520/620: Group Theory

a. Objectives: Basic topics, including semidirect products, wreath products, Sylow subgroups of symmetric groups, actions on sets, semidihedral, dihedral and quaternion groups, p-groups. At the same time: Computational group theory using the computer system GAP.

b. Content: Topics will include the following:

- Free groups, groups acting on trees
- Crystallographic groups
- Coxeter groups
- Finite simple groups
- Representation theory

c. Reading materials include:

- Serre's Linear Representations of Finite Groups
- Alperin and Bell's Groups and Representations.
- I.M. Paramanov, Symmetries in Mathematics.
- Deng, B. and Du, J. et al (2008), Finite Dimensional Algebras and Quantum Groups, Rhode Island, American Mathematical Society.
- Grillet, P.A. (2007), Abstract Algebra, New York, Springer-Verlag N.Y. Inc.

MSCI 571/671: Topics in computational Geometry

a. Objectives: By the end of the course students will be able to

- Describe the main concepts in computational geometry
- design efficient algorithms and data structures to solve geometric problems

b. Content: In the course we will focus on some fundamental problems in computational geometry and algorithmic concepts that can be used to solve them. We will cover: Convex hulls, polygon triangulation, Delaunay triangulations, Voronoi Diagrams, arrangements, projective duality, geometric optimization, linear programming, range searching, point location and other geometric data structures.

c. Reading materials include:

1. Discrete and Computational Geometry, Satyan L. Devadoss and Joseph O'Rourke (2011) Princeton University Press
2. F. Preparata. M. Shamos, Computational Geometry, (1985) Springer Verlag
3. M. de Berg, M. van Kreveld, M. Overmars, O. Schwarzkopf, Computational Geometry: Algorithms and Applications, (1999) Springer Verlag
4. J. O'Rourke, Computational Geometry in C, Second Edition, (1998) Cambridge University Press

MSCI 524/624: Evolutionary graph theory

a. Objectives: By the end of the course, students should be able to



- describe key concepts of evolutionary graph theory
- mathematically analyze discrete-time stochastic processes
- compute the asymptotic runtime of randomized algorithms

b. Content: Evolutionary graph theory is a field at the interface of mathematics and theoretical computer science. It studies discrete-time random processes that model how different entities spread through networks (graphs). Such processes may describe the spread of fake news, memes, or different opinions on social networks, the spread of infectious diseases in human populations, or the spread of genetic mutations in spatially structured biological populations. In this course, we will study several such processes, and we will focus on understanding how the underlying network structure affects their outcomes. Along the way, we will encounter probabilistic concepts such as Markov chains, random walks, or martingales, and also computer science concepts such as graph expansion or asymptotic computational complexity.

c. Reading materials include:

1. Martin A. Nowak (2006). *Evolutionary Dynamics*
2. Thomas H. Cormen (2009). *Introduction to Algorithms*
3. David A. Levin, Yuval Peres (2017). *Markov Chains and Mixing Times*

MSCI 596/696: Mathematical Biology

a. Objectives: At the end of the course the students should be able to apply the principles and tools from mathematics to biological systems.

b. Content: This course will provide an introduction to the use of differential equation models to understand biological processes. Examples will be drawn from population dynamics, infectious diseases, gene expression and pattern formation. The mathematical approach will focus on a dynamical systems approach, understanding dynamics in terms of phase portraits and bifurcations. Students will be able to conduct numerical explorations of the models in the course using Python, but this is not a core component of the course.

Prerequisites: Linear algebra (for linear stability analysis), ordinary differential equations. Exposure to the basics of dynamical systems (phase portraits, attractors, local stability) is helpful but not essential. The course will be as a vehicle for an elementary introduction to stochastic models (master equation, direct simulation, stochastic differential equations), and I often do this as an optional project. The focus is on simulation rather than theory.

c. Reading materials include:

1. J. D. Murray (1989) *Mathematical Biology* Springer-Verlag, Berlin Hiedelberg.
2. Phil Nelson (2003), *Biological Physics* WH Freeman and Co, ISBN 0716743728.
3. Phillips et al (2013) *Physical biology of the cell* by Garland Science, Taylor & Francis Group, LLC
4. Alberts et al (2014) *Molecular biology of the cell* ISBN: 9780815344322.

MSCI 522/622: Cryptography

a. Objectives: Cryptography has become important in today's online world. It allows us to carry out online financial transactions and many similar applications. Cryptography uses mathematics to code and encode messages.

b. Content: In this course, we will compare subsum methods to the older RSA method, and students can pick which algorithm they would like to learn more about. Students will write code for either a subsum method or the RSA method. The subsum problem is an example of a



problem that is useful in cryptography. A simple case might be: you are told that some numbers chosen from the list $\{2, 3, 4\}$ add up to 7. What are the numbers? This problem is not too hard, and we can see that the answer is $3 + 4 = 7$. Probably you solved the problem by trying a few combinations of the numbers in the list. But problems like this can be quite hard if there are for example a hundred large numbers to choose from. We could try using a computer to try all the possible combinations one after the other, but there are a great many such combinations. In the case of a list of a hundred distinct numbers there are 2100 ways to choose a subset of distinct numbers. This is the basis of the subsum problem that is sometimes used in cryptography. Some subsum problems are easy to “crack,” some are not. To use them in cryptography, we choose a list of numbers from a subsum problem that can be hard, and encode messages by adding up suitable subsets of numbers, and then sending the sum that we got as our secret message. Then in order to decode the message we need a private key and a decoding method. In many cases this involves taking exponents in modular arithmetic and then testing the result against a private key. Not all subsum problems are hard, as you see in the example above. So a subsum problem, also called a knapsack problem, has to be properly chosen in order to be hard enough to use in cryptography. There is a method of deciding if a knapsack is hard or easy, called knapsack density. We’ll discuss modular arithmetic, sumset problems in modular arithmetic, and what it means for a knapsack to have a high density or a low density. The density has a lot to do with how hard the problem is. There are special results that are useful in choosing a hard sumset problem. The RSA problem is based on multiplication instead of addition. It uses a lot of number theory, which we’ll explain, and the basic problem used in RSA is how to factor a number into prime factors.

c. Reading materials include:

1. Bach, E. Discrete logarithms and factoring (Ser. Report, no. UCB/csd-84-186). Computer Science Division, University of California. (1984).
2. E. F. Brickell, Solving Low-Density Knapsacks, Advances in Cryptology, Proceedings of Crypto 83, Plenum Press.
3. W. Diffie, M. E. Hellman, New Directions In Cryptography, IEEE Transactions On Information Theory, Vol. II-22, No. 6, November 1976
4. R.H. Cooper, R. Hunter-Duvar, and W. Patterson, A More Efficient Public-Key Cryptosystem Using the Pascal Triangle, ICC ‘89, Boston, June 1989, pp. 1165–1169.
5. R.H. Cooper, Jeff Retallick, and Brent R. Petersen, “A public-key system based on primes and addition”, in Boris Shishkov and Andon Lazarov, eds, ICTRS 2023, Communications in Computer and Information Science, vol 1990, pp. 51-64, Springer, Cham, Switzerland, Dec. 2, 2023
6. D. E. Flath, Introduction to Number Theory, AMS Chelsea publishing (1989)
7. J.C. Lagarias, A. M. Odlyzko, Solving Low-Density Subset Sum Problems, Proc. 24th Annual IEEE Symposium on Foundations of Computer Science (1983), 1–10.
8. S. Rubenstein-Salzedo, Cryptography, Springer (2018)

MSCI 556/656: Numerical Analysis with Python

a. Objectives:

The major concern of numerical analysis is to derive practical algorithms to solve mathematical problems. Even the most sophisticated mathematician can only solve a fraction of real world problems by hand. This may be because of the size of the problem, or the fact that problems can’t be solved analytically. The computer is a necessary tool, for in practice we may encounter problems of enormous size or we may need to solve thousands of intermediate



problems to reach our ultimate goal. Two essential attributes of a practical algorithm are speed and accuracy, and these can lead to conflicting priorities.

In this course we derive a number of algorithms for solving problems in diverse fields of mathematics, focused on ODEs and matrix computations. Along the way we will learn how to balance speed and accuracy by understanding the effects of discretising problems and due to the way computers perform arithmetic calculations. We will emphasise both the theoretical properties of numerical algorithms and practical implementation.

There is a strong focus on modern applications and numerical implementation throughout the course. We will make active use of Python and Jupyter workbooks throughout as a pedagogical tool, to enhance existing programming skills and broaden knowledge of modern areas of applied mathematics, as a way of testing theory, and as an environment for authoring algorithms.

b. Content:

Numerical Schemes for Solving ODEs: Euler's method, Taylor series, Runge-Kutta schemes
Consistency, stability and convergence of numerical methods and order conditions for RK schemes

Applications to solving ODEs and mathematical modelling using Python

Matrix multiplication: Reordering loops. Strassen's method. Matrix powers. Sparsity.

Solving linear systems: Understanding error. Norms. Elimination methods. Iterative methods.

Orthogonality: Inner products. Gramm-Schmidt. Givens rotations and Householder reflections.

The algebraic eigenvalue problem: Power method and shifts. Invariant subspaces. QR iteration.

c. Reading materials include:

1. Numerical Methods for Ordinary Differential Equations by D. F. Griffiths and D. J. Higham <https://doi.org/10.1007/978-0-85729-148-6>
2. Numerical Linear Algebra by L. N. Trefethen and D. Bau <https://doi.org/10.1137/1.9780898719574>
3. Computational Mathematics: An introduction to Numerical Analysis and Scientific Computing with Python by D. Mitsotakis <https://doi.org/10.1201/9781003287292>

MSCI 526/626: Statistical Physics

a. Objectives: By the end of the course students should be able to:

1. define a microstate and macrostate of a model system
2. discuss the concept of entropy and free energy from the view point of statistical mechanics
3. define the Boltzmann distribution and calculate partition functions
4. apply the machinery of statistical mechanics to the calculation of macroscopic properties resulting from microscopic models

b. Content:

Statistical Mechanics is the mathematical theory that underpins our understanding of systems of large ($\sim 10^{23}$) numbers of 'particles' (e.g gases, liquids and solids). It turns out that due to the large number of particles, we can make rather a lot of progress in describing the average properties of these systems. Such large systems are often referred to as macroscopic. The 'particles' or basic entities which make up these systems are referred to as microscopic. Our



goal in this course is the study of such large systems which we can approach in 2 ways - macroscopic and microscopic. An outline of the contents is as follows:

1. Thermodynamics: macroscopic systems, state variables, laws of thermodynamics, engines and fridges, phase equilibria
2. Equilibrium Classical Statistical Mechanics: - ensembles, derivation of thermodynamic quantities, entropy of mixing, ideal gas
3. Dynamical Foundations and aspects of non-equilibrium: - Hamiltonian mechanics, Liouville's equation, Poincaré recurrence, Boltzmann equation and H-theorem
4. Equilibrium Quantum Statistical Mechanics: - quantum statistics, density matrix, boson and fermion systems, Bose-Einstein condensation
5. Modern Topics: simulations, dynamics of fluctuations

c. Reading materials include:

1. Landau & Lifshitz Statistical Physics ISBN: 9780750633727
2. Kittel, Charles. Thermal Physics ISBN 0-471-49030-X
3. Blundell, Stephen; Blundell, Katherine (2006). Concepts in Thermal Physics Oxford University Press. ISBN 978-0-19-856769-1
4. Hagelstein, P. L., Senturia, S. D. and Orlando, T. P. (2004) Introductory Applied Quantum and Statistical Mechanics, Wiley.
5. Reed, T. M. and Gibbons, K. E. (1991) Applied Statistical Mechanics: Thermodynamic and Transport Properties of Fluids, Butterworth-Heinemann Series in Chemical Engineering.
6. Weiss, G. H. (Ed) (2015) Contemporary Problems in Statistical Physics, SIAM.
7. R.K. Pathria, Statistical mechanics, Elsevier 2005.
8. D. Chandler, Introduction to modern statistical mechanics, Oxford 1987.
9. J.R. Dorfman, An introduction to chaos in non-equilibrium statistical mechanics, Cambridge 1999.
10. W. Pauli, Pauli lectures on physics, vol 3 : Thermodynamics and the kinetic theory of gases, Dover 2003, 160 pages.
11. M. Kardar, Statistical physics of particles, Cambridge 2007, 330 pages
12. M. LeBellac, F. Mortessagne and G. Batrouni, Equilibrium and non-equilibrium statistical thermodynamics, Cambridge 2004, 616 pages.

MSCI 532/632: Hyperbolic Geometry

a. Objectives: By the end of the course students should be able to:

- compare different models of hyperbolic geometry
- calculate the hyperbolic distance between points in the hyperbolic plane and the geodesic through them
- prove results in hyperbolic trigonometry and use them to calculate angles, lengths, areas, etc, of hyperbolic shapes
- classify Möbius transformations in terms of their actions on the hyperbolic plane
- define a finitely presented group in terms of generators and relations
- use Poincaré's Theorem to construct examples of Fuchsian groups

b. Content: In this course I will provide an introduction to hyperbolic geometry and the hyperbolic plane. We will study how discrete groups of isometries act on the hyperbolic plane. Hyperbolic space is geometry which has constant negative curvature. Complex hyperbolic



space has homogeneous geometry with variable negative curvature. Hyperbolic space is an important example in Riemannian geometry. In this course, I will introduce real and complex hyperbolic manifolds. I will focus on a series of concrete constructions (both arithmetic and geometric). We will study compact manifolds with metrics locally isometric to these hyperbolic spaces. In addition, we will consider deep connections between the study of these manifolds and other areas of mathematics.

c. Reading materials include:

1. A.F. Beardon, The Geometry of Discrete Groups, Springer, 1983
2. D.B.A. Epstein, Complex hyperbolic geometry in "Analytical and Geometric Aspects of Hyperbolic Space", ed. D.B.A. Epstein, London Mathematical Society Lecture Notes Series 111 (1987), 93–111
3. W.M. Goldman, Complex Hyperbolic Geometry, Oxford University Press, 1999
4. F. Kirwan, Complex Algebraic Curves, L.M.S. Student Texts 23, Cambridge University Press, 1992
5. E.H. Lockwood, A Book of Curves, Cambridge University Press, 1961

MSCI 587/687 Category theory & applications in AI

a. Objectives:

This course combines the pure mathematics field of category theory with modern applications in artificial intelligence. As machine learning methods such as neural networks become more complex, the general framework of category theory is becoming important in describing computational structures.

By the end of the course the students should be able to define fundamental definitions in category theory and use them to make connections between different fields of mathematics, in particular applications in AI.

b. Content:

Many advances in modern mathematics are due to work in category theory which allow results in one field to translate to or heavily inform results in other fields. This course will cover fundamental definitions and the theory of covariant and contravariant functors, small and concrete categories, products and coproducts, limits, and how they are used to create both straightforward and surprising connections between disparate fields of mathematics. Opportunity will be afforded for individuals or small groups to investigate particular relationships of interest.

c. Reading materials include:

1. "Category Theory: An Introduction" Herrlich and Strecker
2. Notes on Category Theory with examples from basic mathematics by Paolo Perrone
3. Conceptual Mathematics: A First Introduction to Categories by F. William Lawvere and Stephen H. Schanuel
4. Basic Category Theory (2014) Tom Leinster, ISBN 9781107044241
5. Category Theory in Context: <https://emilyriehl.github.io/files/context.pdf>

